The More Efficient, the More Vulnerable!

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Abstract

We extend the limited arbitrage model of Shleifer and Vishny (1997) to an intertemporal model while simplifying a funding cost structure. The model implies that the equilibrium price is more volatile during a crash than during a tranquil market period. More importantly, a seemingly more efficient market is more vulnerable to a crash and shows more extreme tail volatility and a larger difference between tail volatility and non-tail volatility. We empirically examine such implications in a U.S. interest rate swap market. The mean-reversion speeds of slope and butterfly spreads between swap yields are strongly associated with tail behavior of those spreads, which is in compliance with our model.

1 Introduction

It is the calm and silent water that drowns a man.

–Ghanaian proverb

No arbitrage is one of the fundamental concepts in finance. Theoretically speaking, no arbitrage is achieved when the prices of all security are determined such that any state security with a dollar payoff a.k.a. 'Arrow-Debreu security' is positively priced. Harrison and Kreps (1979) and Harrison and Pliska (1981) show that given the well-known one-to-one correspondence between the state price and the equivalent martingale measure, no arbitrage condition is satisfied when the equivalent martingale measure exists and it is strictly positive across all states. In contrast, the law of one price, which is better known, indicates that any assets with the same payoff should be priced the same. Harrison and Kreps (1979) show that the law of one price is a subset condition of no arbitrage condition and it requires simply the existence of equivalent martingale measure without restrictions on its signs.

The arbitrage transactions, which attempt to monetize the discrepancy of a particular asset's market price from its fundamental value, play a critical role of policing and eliminating such discrepancy. The textbook version of the efficient market hypothesis argues that such arbitrage transactions are strong enough to eliminate any market disequilibrium *instantaneously*. Kyle (1985) defines the market resilience, a facet of liquidity, as the speed with which prices revert to their equilibrium level after a large shock in the transaction flow. The activity of arbitrage transactions *per se* is the key determinant of the market resilience; the more active the arbitrage trades, the more resilient the market becomes. Therefore, textbook efficiency implies the 'infinite' speed of market resilience, which is durable only if arbitrage transactions can be implemented in a perfect market, i.e., no market frictions.

Shleifer and Vishny (1997) argue that such textbook arbitrage is at odds with reality. They theoretically explore the reason arbitrage fails to eliminate disequilibrium and show that aribtrage becomes ineffective in extreme circumstances, when market prices diverge far from fundamental values. Their study suggests two components as the cause of limits to arbitrage. The first is friction. Gromb and Vayanos (2010) classify market frictions into three categories: short-sale constraints, leverage constraints and equity capital constraints. For example, arbitragers would not be able to fully exploit arbitrage opportunities due to an increase in borrowing costs or other borrowing constraints. Furthermore, borrowing capacity is positively associated with collateral values, which are an outcome of past trades. Therefore, disequilibrium becomes more pronounced after arbitragers experience capital losses from existing positions, which abates their borrowing capacity. The second component is demand shock. Arbitragers tend to rely upon short-term funding with limited borrowing capacity. As such, when noise traders deviate market prices farther from their fundamental values, they may experience interim losses, which may not be recoverable due to a shrinkage on leverage capacity. The arbitragers recognize such risk and would strategically *a priori* downsize their trade size or even shun the trade after all. If the arbitragers are risk averse, this risk further deters the arbitragers from conducting arbitrage transactions.

In this paper, we add a new insight to the 'limits to arbitrage' literature by introducing an intertemporal aspect of arbitrage. Specifically we extend a simplified version of Shleifer and Vishny model by adding one more time period. Such an extension is non-trivial due to the nature of path-dependency through wealth effect in endogenous variables such as leverage ratios and market prices. The results deliver a number of interesting implications for security prices.

Firstly, we show that the past performance does not make much impact on the security price in a mediocre state. In contrast, its impact is pronounced in a 'crash' state. In a crash state, the arbitrager has to take a substantially high leverage to take advantage of mispricing. However, this is the very state where the leverage constraint is most binding. How much leverage the arbitrager should take depends upon how much capital loss or gain has been cumulated from past trades. Consequently, the performance of the arbitrager preceding the crash, has the most critical impact on the security price. This indicates that the volatility of security price during the crash should be greater than the volatility during the tranquil time.

Secondly, when the evolution of states entails a higher probability of reversion from a disequilibrium state to a normal state, the security price is more likely to plunge in a crash state. That is, when the security price shows a stronger reversion to a normal state during a tranquil time, it is more vulnerable to a crash. This is a striking result because a seemingly more efficient market under a normal market condition would be more frail and more likely to be dismantled during times of crises.

Thirdly, we also find that when the security price shows a stronger reversion to a normal state during a tranquil time, its volatility during times of crises is larger. That is, the crash time volatility difference between a seemingly more efficient security and a seemingly less efficient security is much larger than the tranquil time volatility difference betwen the two.

We empirically investigate the aforementioned implication of our model using a U.S. interest rate swap market. Using thirteen different tenors from one year to thirty years, we construct 78 slope spreads and 286 butterfly spreads. These are the most popular instruments for relative value trades a.k.a. 'fixed income arbitrage' adopted by hedge funds and proprietary trading desk of global investment banks. These spread trades attempt to monetize the abnormal widening or narrowing of the spread among two or three swap legs (even four legs, which is called 'box' trade, mostly constructed from two asset swap spreads). Because the spreads contain simultaneous long and short positions on multiple legs, they tend to cancel out systematic risk such as duration risk. However, the payoff profile of the fixed income arbitrage is, in general, very small so that leverage is widely employed to magnify potential gains, typically five to fifteen times the asset base's value.

An empirical analysis based on the swap data from July 23, 1998 to May 11, 2017 shows an overall compliance with the theoretical predictions. First, we find that the mean-reversion speed of the spreads is positively associated with the extreme movement measures of the spreads. Specifically, the spreads with higher mean-reversion speed tend to have the greater absolute z values of extreme percentiles such as 0.05%, 1% and 2% and also correpsondingly 98%, 99% and 99.5%. When we use the corresponding expected shortfall risk values as an alternative proxy for extreme movement (crash values), the result remains almost the same, albeit stronger. As a result, the kurtosis of distribution of the spreads demonstrate strong positive relations with the mean-reversion speeds. These results imply that the more seemingly efficient spreads with faster mean-reversion are more susceptible to extreme change in their values, thereby more vulnerable to a crash, which our model predicts.

In addition, we investigate whether the mean-reversion speed is also positively associated with the conditional volatility upon the aforementioned percentiles. The empirical results demonstrate a strong positive relationship between the two, which supports our model, which predicts that the spreads with stronger mean reversion speed should be more likely to exhibit higher volatility at crash states. In contrast, we detect no relationship or a much weaker relationship between the mean-reversion speed and non-tail volatility.

This paper is organized as following. In Section 2, we build up a four-period aribtrage transaction model wherein a sequence of equilibrium prices is determined and thus we can analyze the impact of limits of arbitrage on the efficiency and the vulnerability. The empirical analysis on the major predictions of the model using the U.S. interest rate swap data is examined in Section 3. Proofs of propositions are deferred to Appendix. Section 4 concludes.

2 The Model

The basic structure of our model follows Shleifer and Vishny (1997). We extend their model by adding more states and time periods in order to explore the path-dependency of the market price. We also simplify the structure of funding cost to avoid corner solutions without losing economic intuitions underlying their model. We consider an asset market the fundamental value of which is assumed to be V. In this market, there are two market participants: noise traders and an arbitrager. The noise traders trade for liquidity reasons not related to the asset's fundamental value thereby making deviation of its market price from the fundamental value. Without loss of generality, we assume that the amount of deviation is random but non-positive; that is, noise traders may experience pessimistic shocks as in Shleifer and Vishny (1997).[1] In contrast, the arbitrager, who knows the fundamental value, attempts to monetize the mispricing of the asset which is triggered by the noise traders.

There are four time periods: $t = 0$, 1, 2 and 3. The fundamental value of the asset is V for all t, which only the arbitrager recognizes. In contrast, the noise traders may trigger negative shocks to the market price. The state space of negative noise trader shocks is illustrated in Figure 1. Therein the amount of shock at $t = 0$ is $-\frac{1}{2}S$ ($S > 0$), which is known to the arbitrager, but the noise trader shocks in the suqsequent periods are uncertain. For example, the states at $t = 1$ is binomial such that the amount of shock is either zero with probability of $(1 - q)$ or $-S$ with probability of q where $0 < q < 1$. Going forward to $t = 2$, the state space is trinomial; the amount of shock is 0, $-S$ and $-2S$ with probability of $1 - q$, $\frac{1}{2}q$ and $\frac{1}{2}q$ respectively, regardless of the state at $t = 1$. So we implicitly assume path-independence of conditional probability of each state. For example, the occurrence of zero noise trade shock is $1 - q$ either when the state at time $t = 1$ is the first node or when it is the second node. The amount of noise shock is equal to 0 at $t = 3$ for sure, and hence the market price equals V.

Overall, the structure of state space is similar to that of Shleifer and Vishny (1997). For comparison, Figure 1 exhibits the state space assumed in their model as thick lines, which is nested in our state space. As such, we add one more time period and one more state at $t = 2$. We consider such an extension for two reasons. First, by adding more states at $t = 2$, we can separate 'crash' shock, $-2S$ from reasonably 'accommodative' shock $-S$. Note that the initial amount of shock is $-\frac{1}{2}S$ at $t = 0$. As such, the incremental amount of shock to the arbitrager in the second state at $t = 2$ is only $-\frac{1}{2}S$. In contrast, if the third

[1] The main result of our analysis holds in a symmetric way when the noise traders may experience optimistic shocks, as long as the funding structure is symmetric between long and short positions.

state occurs, the arbitrager has to bear $-\frac{3}{2}S$, which is three times larger. In addition, the amount of shock, $-S$, may occur at $t = 1$ and $t = 2$ whereas the shock, $-2S$, comes into being only once over the entire periods in our analysis. As such, the state of $-2S$ can be thought of as a tail risk in terms of its magnitude as well as its probability of occurrence. In contrast, the state of $-S$ can be regarded as a shock during a tranquil time period.

Second, adding one more time period enables us to analyze the impact of past performance on the market price, as will be shown below. For example, the second state at $t = 2$ can be reached either through the first state or the second state at $t = 1$. If it is reached via the first state at $t = 1$, the arbitrager earns profit at $t = 1$. On the contrary, she makes a loss at $t = 1$ if it is reached via the second state. Then the intuition of Shleifer and Vishny (1997) kicks in. They assume that the availabile investment amount of the arbitrager is an increasing function of her past return and they call this 'performance-based arbitrage'. In our model, instead, we assume that the funding cost is proportional to the amount of leverage. Consequently, if the arbitrager earned positive profit in the prior period, she affords to take more leverage because the funding cost is cheaper and vice versa. Thus, the arbitrage is dependent upon her past performance, which is similar to Shleifer and Vishny. As shown below, our assumption is easier to analyze with and we can avoid corner solutions under some regularity conditions.

Assumption 1 (Funding Cost) *We assume the following funding cost structure. Suppose that the arbitrager's wealth is W and borrows $L \geq -W$. Then, the funding rate is assumed to be proportional to leverage ratio, $\psi = \frac{L}{W}$; e.g,*

$$c(\psi) = r + \phi\psi 1_{\psi>0}, \tag{1}$$

where r is the risk-free rate and $1_{\psi>0}$ is an indicator variable with a value of 1 if leverage is employed and 0 otherwise. $\phi > 0$ is a sensitivity of the funding rate to the leverage. $\psi \in [-1, \infty)$. Without loss of generality, we assume $r = 0$.

Now let's solve for equilibrium prices at each state and each time. To do so, we first assume that the arbitrager is a representative one and behaves as a price taker. We call her 'schizophrenic arbitrager' following the term raised by Hellwig (1980) in the sense that she takes the equilibrium price as given despite the fact that her own transactions influence that price. In addition, we assume that she attempts to maximize her expected wealth in the next period as opposed to her expected terminal wealth; she is a 'myopic arbitrager.'

In the second model, we avoid these undesirable features of the schizophrenic arbitrager by having her take into account the effect her demand has on that equilibrium price. Therein she strategically adjusts her demand or equivalently the leverage ratio to her benefit to maximze her terminal wealth. So we call her 'strategic arbitrager.' In reality, the traders of hedge funds and global investment banks are sophisticated enough to digest the potential impact of her trades on the market price. In that sense, the strategic arbitrager model seems to better reflect the real world.

However, the literature on limited arbitrage including our model assumes that the mispricing of the asset price will be corrected surely at the terminal date of the analysis. Of course, the asset price will converge to its fair value at its redemption date, but, in reality, it might take an extensive amount of time to converge. Thus it may be difficult for her to make a strategic arbitrage intertemporally. In addition, an arbitrager may be nothing more than an atom in a continuum of arbitragers and she may not recognize that she belongs to this huddled mass of arbitragers whose aggregate demand impacts the price. Therefore, the 'strategic' capacity assumed in the strategic arbitrager model may be overvalued and fails to reflect the real picture of arbitrage. Combining these two concerns, we presume that the real world arbitragers are somewhere in the middle. This is the reason we investigate both models altogether.

2.1 Schizophrenic Arbitrager

In this section, we assume that the risk-neutral arbitrager behaves as a price taker. Due to the assumed structure of funding cost, the equilibrium price and other endogenous variables are path-dependent via a change in wealth. As is the case with most path-dependent intertemporal equilibrium, we are, unfortunately, not able to analytically solve the equilibrium. Even a numerical solution is not trivial and we need to combine a backward induction with a forward deduction to solve the equilibrium.

We first explore a solution for leverage ratio,ψ, given the structure of funding cost. At a particular state i at time t, s_{ti} along with W_t, the arbitrager maximizes her expected wealth at $t+1$ such that

$$\text{Max}_{\{\psi_{ti}\in[-1,\infty)\}} E[W_{t+1}|s_{ti}, W_{ti}] = W_t \left[(1+\psi_{ti}) \frac{E(P_{t+1}|s_{ti})}{P_{ti}} - \psi_{ti}(1+\phi\psi_{ti}1_{\psi_{ti}>0}) \right]. \quad (2)$$

The following proposition summarizes the optimal leverage ratio.

Proposition 1 *The arbitrager's optimal leverage ratio is*

$$\psi^*_{ti} = \begin{cases} \frac{\frac{E(P_{t+1}|s_{ti})}{P_{ti}}-1}{2\phi} > 0 & \textit{if } \frac{E(P_{t+1}|s_{ti})}{P_{ti}} > 1 \\ \in [-1,0] & \textit{if } \frac{E(P_{t+1}|s_{ti})}{P_{ti}} = 1 \\ -1 & \textit{else} \end{cases} \quad (3)$$

However, the above optimal leverage ratio has an undesirable feature that an equilibrium may not exist due to a kink in the cost function. For example, suppose that we compute the interior solution, $\psi^*_{ti} = \frac{\frac{E(P_{t+1}|s_{ti})}{P_{ti}}-1}{2\phi}$. Note that the market clearing condition yields $P_{ti} = V - S_{ti} + W_{ti}(1+\psi^*_{ti})$. Suppose that the resulting value of P_{ti} given ψ^*_{ti} leads to $P_{ti} > E(P_{t+1}|s_{ti})$. As a response, the arbitrager does not take any investment in the security, i.e., $\psi_{ti} = -1$. Then, a new market price $P_{ti} = V - S_{ti}$ could be smaller than $E(P_{t+1}|s_{ti})$, which, in turn, validates a positive leverage ratio. Consequently, the equilibrium price and the optimal leverage ratio could not be compliant with each other and thus the equilibrium may not exist.

A similar problem occurs when $\frac{E(P_{t+1}|s_{ti})}{P_{ti}} = 1$. Since the return on investment is identical between the risk-free asset and the risky asset, the arbitrager would be indifferent between the two assets and thus $\psi^*_{ti} \in [-1, 0]$. However, $P_{ti} = V - S_t + W_{ti}(1+\psi^*_{ti})$ and thus the price itself varies with the difference choice of ψ^*_{ti}. Again, there is no equilibrium which supports the price and the leverage ratio (demand for the asset).

This non-existence of equilibrium is driven by the fact that the optimal ψ^*_{ti} depends on P_{ti} through three channels. First, its interior solution is a direct function of P_{ti}. Second, the market clearing condition designates a relatioship between ψ_{ti} and P_{ti}. Finally, what kind of solution for ψ_{ti} should be adopted is determined by the inequality condition, $E(P_{t+1}) \lesseqgtr P_{ti}$ and that condition itself entails the price, P_{ti}. In general, the first two conditions are sufficient for the existence of the equilibrium. However, the last channel in our model behaves as a sort of overidentifying restriction and the equilibrium may not exist.

The easiest way of ensuring the existence of equilibrium is to impose restrictions of structural parameters which preclude the occurrence of $\frac{E(P_{t+1}|s_{ti})}{P_{ti}} \leq 1$. Then we always have an interior solution for ψ and thus the equilibrium. First, we assume $0 < q < \frac{1}{2}$ to ensure that the expected return of the asset in each of four nodes (the nodes subject to pessimistic shock) is strictly positive in the absense of arbitrage.[2] Further, we impose the boundary condition on S.

Assumption 2 (Boundary Conditions) *We impose the following boundary condition on the pessimistic noise shock, S:*

$$\frac{2W_0}{1-2q} < S < \overline{S} \tag{4}$$

[2]At $t = 0$, the expected return **in the absence of arbitrage** is positive if $E(P_1) = (1-q)V + q(V - S) > P_0 = V - \frac{1}{2}$. This yields $q < \frac{1}{2}$. Similarly, at s_{12}, the expected return is positive if $E(P_2|s_{12}) = (1-q)V + \frac{1}{2}q(V-S) + \frac{1}{2}q(V-2S) > P_{12} = V - S$, which leads to $q < \frac{2}{3}$. Combining the two, $q < \frac{1}{2}$.

where

$$\overline{S} \simeq \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} \leq \frac{V}{2} \tag{5}$$

Below, we show that under the boundary conditions, we can avoid corner solutions. First, we begin with the left inequality condition in Assumption 2.

Proposition 2 *If* $\frac{2W_0}{1-2q} < S$*, the arbitrager takes leverage at every state subjec to negative noise shock; i.e.,* $\psi_{ti} > 0$ *for all* s_{ti} *with* $S_{ti} > 0$*.*

Proposition 2 states that if the magnitude of pessimistic shock is greater than a certain amount, the schizophrenic arbitrager always uses leverage.

Another issue to deal with is negative wealth. Suppose that the *ex-post* path of states is s_{12} at $t = 1$ and s_{23} at $t = 2$. Then the arbitrager makes a loss twice in a row and her wealth at s_{23} might be negative; i.e., the fund collapses. The collapse of the fund itself may be an interesting topic, but how to deal with it is related to the liquidation process of a hedge fund. To prevent it *a priori*, most of hedge funds is equipped with their own internal risk management regulation such that if the fund loses more than a certain percentage, the fund itself will be liquidated. This kind of internal risk control is asserted by a contract between the fund and its investors. This issue itself is an interesting topic for further analysis but is beyond the scope of this paper.[3] So we preclude a collapse of the fund by imposing a certain restriction on structural parameters. The approximate restrictions are summarized in Proposition 3, which validates the upper bound in Assumption 2. From here on, we simplify notations in expressing path-dependency such that $\psi_{s_{t+1i}|s_{tj}}$ for $(\psi_{s_{t+1i}}|s_{tj})$. For example, $\psi_{22|12}$ refers to the leverge ratio at s_{22} when s_{22} is realized via s_{12}. Similar subscripts will be used to express path-dependency in other relevant variables.

Proposition 3 $W_{23|12} > 0$ *if*

$$S < \overline{S} \simeq \begin{cases} \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} & \text{if } \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \lessgtr 0. \\ \frac{V}{3} & \text{if } \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi = 0 \end{cases}$$

Thus, the (approximate) upper bound on S proposed in Assumption 2 guarantees the solvency of the arbitrager and hence non-degenerate solution for the equilibrium. In summary, under the boundary conditions, the arbitrager always employs leverage except when

[3] See Ahn, Kim and Seo (2017) for a fund run driven by such a contract coupled with other frictions.

negative noise shock does not exist, i.e., $S_{ti} = 0$, and her fund is ensured to be solvent. Consequently we can alway find an interior solution to the equilibrium.

Proposition 4 *Under the boundary conditions on S in Assumption 2, which ensure the existence of interior solutions to ψ^*_{ti} and positive wealth of the arbitrager, the equilibrium price at s_{ti} is*

$$P_{ti} = \frac{b_{ti} + \sqrt{b_{ti}^2 + 8\phi W_{ti} E(P_{t+1}|s_{ti})}}{4\phi} \tag{6}$$

where $b_{ti} = 2\phi(V - S_{ti}) + (2\phi - 1)W_{ti}$.

Proof: Plugging the interior solution to ψ^*_{ti} in Proposition 1 into the following market clearing condition

$$P_{ti} = V - S_{ti} + W_{ti}\left(1 + \psi^*_{ti}\right),$$

and solving for P_{ti} yields the desired result. □

To solve for the equilibrium, we have to note that the equilibrium price is inevitably path-dependent. The market clearing condition in each node is $P_{ti} = V - S_{ti} + W_{ti}(1 + \psi_{ti})$, and thus the price is affected by W_{ti}.[4] Below we discuss how to find a solution, numerically. Below we begin with the equilibrium from $t = 2$ and move back to $t = 0$.

Equilibrium at $t = 2$

Following Proposition 1 and Proposition 3, we can derive the equilibrium price and the optimal leverage.

- **$i = 1$:**
  Since $S_{21} = 0$, the asset is not subject to mispricing and thus the arbitrager becomes dormant.[5]

[4] However, it does not necessarily mean that the intuition underlying our model is non-Markovian. It is an outcome of the fact that the state space is governed by two state variables: s_{ti} and W_{ti}. We can extend the number of time periods and the number of states enough to make the conditional expected return identical (in the absence of arbitrage) at the same state across different time. Then the optimal leverage ratio at the same state with the same wealth will always be the same since she is myopic.

[5] She may invest a part of her wealth in the asset, e.g., $-1 < \psi_{21} < 0$ since the interest rate is zero. Then the resulting equilibrium price is greater than its fair value on the back of the arbitrager's excess demand. The equilibrium does not exist since the expected return of the asset becomes negative. To prevent it, we assume that she does not invest in the asset when there is no noise shock. This assumption is sensible in the sense that the arbitrager monitors multiple asset markets to detect arbitrage opportunities. When a particular market does not deliver an arbitrage opportunity, she may invest her wealth in arbitrage opportunities somewhere else.

The optimal leverage ratio and the equilibrium price are

$$\begin{aligned} \psi_{21|1j} &= 0 \\ P_{21|1j} &= V \end{aligned} \qquad \forall j = 1, 2 \tag{7}$$

- $\boldsymbol{i = 2, 3}$

  Following Proposition 1 and Proposition 4, we can obtain the following optimal leverage ratio and equilibrium price:

$$\psi_{2i|1j} = \frac{\frac{V}{P_{2i|1j}} - 1}{2\phi} \tag{8}$$

$$P_{2i|1j} = \frac{b_{2i|1j} + \sqrt{b_{2i|1j}^2 + 8\phi W_{2i|1j} V}}{4\phi} \tag{9}$$

$$\text{where } b_{2i|1j} = 2\phi(V - S_{2i}) + (2\phi - 1)W_{2i|1j}$$

$$S_{2i} = \begin{cases} S & \text{if } i = 2 \\ 2S & \text{if } i = 3 \end{cases}$$

  for $i = 2, 3$ and $j = 1, 2$. Note that they are the functions of two state variables: negative noise shock, S_{2i}, and the wealth of the arbitrager, $W_{2i|1j}$.

Equilibrium at $t = 1$

At $t = 1$, there are two states, $j = 1$ and $j = 2$.

- $\boldsymbol{j = 1:}$

  Since $S_{11} = 0$, the optimal leverage ratio and the corresponding equilibrium price are

$$\begin{aligned} \psi_{11} &= 0 \\ P_{11} &= V \end{aligned} \tag{10}$$

- $\boldsymbol{j = 2:}$

  Again, following Proposition 1 and Proposotion 3, the optimal leverage ratio and the equilibrium price are

$$\psi_{12} = \frac{\frac{E(P_{2|12})}{P_{12}} - 1}{2\phi} \tag{11}$$

$$P_{12} = \frac{b_{12} + \sqrt{b_{12}^2 + 8\phi W_{12} E(P_{2|12})}}{4\phi}$$

$$\text{where } E(P_{2|12}) = (1 - q)V + \frac{q}{2} P_{22|12} + \frac{q}{2} P_{23|12}$$

$$b_{12} = 2\phi(V - S) + (2\phi - 1)W_{12}$$

However these solutions are not complete yet because we do not know $P_{22|12}$ and $P_{23|12}$, which are the functions of $W_{22|12}$ and $W_{23|12}$. However, $W_{22|12}$ and $W_{23|12}$ are also the functions of $P_{22|12}$ and $P_{23|12}$ respectively. Underlying intuition is as following: the price, for example, $P_{22|12}$, depends on how wealthy the arbitrager is then $(= W_{22|12})$, and also how much leverage she employs $(=\psi_{22|12})$. But, in turn, the wealth of the arbitrager $(= W_{22|12})$ is affected by the realized price, $P_{22|12}$. Futhermore, the optimal leverage ratio $\psi_{22|12}$ is affected by $P_{22|12}$. Unfortunately we are not able to analytically solve these simultaneous nonlinear problems. As such, we numerically solve for $W_{22|12}$ as a function of W_{12}, ψ_{12}, P_{12} only, not contemporaneous variables such as $\psi_{22|12}$ and $P_{22|12}$. A similar problem applies to $W_{23|12}$.

First, let us begin with $W_{22|12}$. Note that

$$W_{22|12} = W_{12}\left[(1+\psi_{12})\frac{P_{22|12}}{P_{12}} - \psi_{12}(1+\phi\psi_{12})\right]. \tag{12}$$

Plugging (9) into (12) yields

$$W_{22|12} = W_{12}\left[(1+\psi_{12})\frac{b_{22|12} + \sqrt{b_{22|12}^2 + 8\phi W_{22|12} V}}{4\phi P_{12}} - \psi_{12}(1+\phi\psi_{12})\right].$$

Note that $b_{22|12}$ itself is a function of $W_{22|12}$ as well since $b_{22|12} = 2\phi(V-S) + (2\phi - 1)W_{22|12}$. Solving the above equation for $W_{22|12}$ leads to a quadratic equation for $W_{22|12}$. Its solution is

$$W_{22|12} = \frac{-\mathbb{B}_{22|12} + \sqrt{\mathbb{B}_{22|12}^2 - 4\mathbb{A}\,\mathbb{C}_{22|12}}}{2\mathbb{A}} \tag{13}$$

where

$$\begin{aligned}
\mathbb{A} &= \eta^2 - (2\phi-1)^2 \\
\eta &= \frac{4\phi P_{12}}{(1+\psi_{12})W_{12}} - (2\phi - 1) \\
\mathbb{B}_{22|12} &= 2\eta\, \upsilon_{22|12} - 4\phi(2\phi-1)(V-S) - 8\phi V \\
\mathbb{C}_{22|12} &= \upsilon_{22|12}^2 - 4\phi^2(V-S)^2 \\
\upsilon_{22|12} &= \frac{4\phi P_{12}}{(1+\psi_{12})}\psi_{12}(1+\phi\psi_{12}) - 2\phi(V-S).
\end{aligned}$$

A similar analysis shows that $W_{23|12}$ is

$$W_{23|12} = \frac{-\mathbb{B}_{23|12} + \sqrt{\mathbb{B}_{23|12}^2 - 4\mathbb{A}\,\mathbb{C}_{23|12}}}{2\mathbb{A}} \tag{14}$$

where

$$\begin{aligned}
\mathbb{B}_{23|12} &= 2\eta\, \upsilon_{23|12} - 4\phi(2\phi-1)(V-2S) - 8\phi V \\
\mathbb{C}_{23|12} &= \upsilon_{23|12}^2 - 4\phi^2(V-2S)^2 \\
\upsilon_{23|12} &= \frac{4\phi P_{12}}{(1+\psi_{12})}\psi_{12}(1+\phi\psi_{12}) - 2\phi(V-2S).
\end{aligned}$$

Note that $W_{22|12}$ and $W_{23|12}$ are expressed as the functions of W_{12}, ψ_{12} and P_{12}.[6] A remaining task is to solve the optimal ψ_{12} and the equilibriium price P_{12} given W_{12} by the following procedure of iteration. Beginning with an initial value of ψ_{12}^0, the k_{th} iteration is composed of

(1) Take the given value of ψ_{12}^k

(2) Compute $P_{12}^k = V - S + W_{12}(1+\psi_{12}^k)$

(3) Given W_{12}, ψ_{12}^k, P_{12}^k, compute $W_{22|12}^k$ and $W_{23|12}^k$ using (13) and (14) respectively.

(4) Compute $P_{22|12}^k$ and $P_{23|12}^k$ using (9).

(5) Put $P_{22|12}^k$ and $P_{23|12}^k$ into (11) and calculate an updated value of ψ_{12}^{k+1}.

(6) Go to step (1) and iterate the procedure until $\psi_{12}^{k+1} = \psi_{12}^k$.

Equilibrium at $t = 0$

So far we express the optimal leverage ratios, the equilibrium prices and the arbitrager's wealth as the functions of W_{12}. We use W_{12} as an intermediary variable which delivers 'consistency' required for the equilibrium. Here given the initial wealth of the arbitrager, W_0, we use a forward deduction analysis. Specifically, with an initial value of W_{12}^0, we follow the following interative procedure:

(1) Take the given value of W_{12}^k.

(2) Given W_{12}^k, find a solution for P_{12}^k from the iterative numerical procedure in the above at $t = 1$.

(3) Compute P_0^k using

$$\begin{aligned}
P_0^k &= \frac{b_0 + \sqrt{b_0^2 + 8\phi W_0 E(P_1)}}{4\phi} \\
\text{where } E(P_1) &= (1-q)V + qP_{12}^k \\
b_0 &= 2\phi(V - \frac{1}{2}S) + (2\phi - 1)W_0
\end{aligned}$$

[6] Quick aside, $\mathcal{B}_{23|12}^2 - 4\mathcal{A}\,\mathcal{C}_{23|12} > 0$ is the exact condition for $W_{23|12} > 0$ whereas the upper bound on S in Assumption 2 is an approximatation.

(4) Compute the optimal leverage ratio given P_{12}^k:

$$\psi_0^k = \frac{\frac{(1-q)V+qP_{12}^k}{P_0^k} - 1}{2\phi}$$

(5) Compute a newly updated value of W_{12}^{k+1}:

$$W_{12}^{k+1} = W_0 \left[(1+\psi_0^k)\frac{P_{12}^k}{P_0^k} - \psi_0^k(1+\phi\psi_0^k) \right]$$

(6) Go to (1) and iterate until $W_{12}^{k+1} = W_{12}^k$.

(7) After convergence, compute $W_{11} = W_0 \left[(1+\psi_0^*)\frac{V}{P_0^*} - \psi_0^*(1+\phi\psi_0^*) \right]$. At s_{11}, the arbitrager is dormant so that $W_{21|11} = W_{22|11} = W_{23|11} = W_{11}$. Then, using (8) and (9), compute $\psi_{22|11}$, $\psi_{23|11}$ along with $P_{22|11}$ and $P_{23|11}$.

2.2 Strategic Arbitrager

Unlike the schizophrenic arbitrager, the strategic arbitrager is assumed to recognize that her arbitrage transaction affects the price. Thus, she takes into account such an impact on the price when she decides the leverage ratio. In addition, she maximizes her expected terminal wealth, $E(W_3)$, thereby not being myopic. Thus she may intentionally downsize her leverage even at a state with a fairly good arbitrage opportunity in anticipation of bigger chance in the next period. All told, she can strategically adjust her leverage ratios over time. Before we discuss how to solve the optimal leverage ratios and the equilibrium price, let's explore its economic implication.

At time t and state i, the arbitrager solves the same objective function in (2), when her intertemporal concern is ignored. However, even in such a case, its first-order condition becomes

$$\frac{\partial E[W_{t+1}|s_{ti}]}{\partial \psi_{ti}} = W_{ti} \left[\frac{E(P_{t+1}|s_{ti})}{P_{ti}} + \frac{\partial \{E(P_{t+1}|s_{ti})/P_{ti}\}}{\partial \psi_{ti}}(1+\psi_{ti}) - (1+2\phi\psi_{ti}) \right] = 0. \quad (15)$$

In the square bracket, the sum of the first two terms is marginal revenue (MR) of the arbitrage transaction and the last term is its marginal cost (MC). Breaking down the marginal revenue, the first term is the expected return of the asset. The critical term is the second term. This reflects how much the expected return *per se* changes when the leverage ratio increases.

Proposition 5 *At each state, in the neighborhood of the schizophrenic equilibrium, the strategic arbitrager takes a less amount of leverage than her schizophrenic counterpart:*

$$\psi_{ti}^{strategic} > \psi_{ti}^{schizo}.$$

Proposition 5 states that the strategic investor takes a less aggresive leverage by taking into account the effect her demand has on the equilibrium price. This shares the same intution underlying the monopolistic producer. In a rational expectation model under asymmetric information, Kyle(1989) demonstrates that the equilibrium prices reveal less information than those in the presence of the schizophrenia problem. We share the same economic intuition.

The strategic arbitrager also distinguish herself from the schizophrenic arbitrager by maximizing her expected terminal wealth at $t = 3$. In contrast, the schizophrenic arbitrager is *myopic* and maximizes her expected wealth in the next period. Her optimization problem is

$$\begin{aligned}\max_{\{\psi\}} E(W_3) &= (1-q)^2 W_{3|21|11} + \frac{1}{2}(1-q)qW_{3|22|11} + \frac{1}{2}(1-q)qW_{3|23|11} \\ &\quad + q(1-q)W_{3|21|12} + \frac{1}{2}q^2 W_{3|22|12} + \frac{1}{2}q^2 W_{3|23|12}, \end{aligned} \tag{16}$$

where

$$\{\psi\} = \{\psi_0, \psi_{11}, \psi_{12}, \psi_{21|11}, \psi_{22|11}, \psi_{23|11}, \psi_{21|12}, \psi_{22|12}, \psi_{23|12}\}.$$

It is clear that at time 0, the strategic arbitrager strategically determines what she will do at each state in the future. In that context, one thing worthy of mentioning is that the strategic arbitrager model is free from the potential non-existence of equilibrium that the schizophrenic arbitrager model suffers from. The stategic arbitrager can choose $-1 < \psi_{ti} < 0$ optimally. Unlike the schizophrenic arbitrager, she recognizes the impact of her position on the asset price. Therefore she is always able to make **positive** the expected return on the asset. In contrast, the schizophrenic arbitrager may use high leverage enough to push down the expected return to a negative value since she does not recognize such an impact. Thus the strategic arbitrager model does not need a lower boundary condition documented in Assumption 2.

Appendix B describes in detail how to construct and solve the optimization problem in (16).

2.3 The Properties of the Equilibrium

In this section, we discuss the equilibrium and its comparative statics in our model. We first investigate a typical example of the schizophrenic arbitrager model along with the strategic arbitrager model. By doing so, we are able to sort out unique features delivered by each model. Then, we will move onto a comparative static analysis focusing particularly on q, the probability of negative noise shock. To do so, we use the following values of structural parameters:

$$V = 1, \quad W_0 = 0.05 \;\; S = 0.25 \quad \phi = 0.1 \text{ and } q = 0.05.$$

$q = 0.05$ means that each state exposed to negative shock has a 95% chance of reverting to the fair value. $\phi = 0.1$ is that if the arbitrager borrows 100% of his wealth, its funding rate is 10%. It is easy to check that the combination of the above parameter values satisfy the boundary conditions in Assumption 2, which ensures positive leverage ratios and positive wealth of the arbitrager. The numerical solutions to the optimal leverage ratio, the equilibrium price and the resulting wealth of the arbitrager are illustrated in Figure 2(a) and Figure 2(b) for the models of a schizophrenic arbitrager and a strategic arbitrager respectively. We first investigate the schizophrenic arbitrager model and then discuss the stategic arbitrager model for comparision.

2.3.1 The Schizophrenic Arbitrager Model

First we examine the equilibrium of the schizophrenic arbitrager model illustrated in Figure 2(a)

Equilibrium at $t = 0$:

At $t = 0$, the arbitrager borrows 27.99% of her wealth ($=\psi_0$).The resulting equilibrium price ($=P_0$) is 0.939. In the absence of her arbitrager transaction, the price would be 0.875 $\left(= V - \frac{1}{2}S = 1 - \frac{1}{2}0.25\right)$. Thus, her investment itself raises the market price by 0.064.

Equilibrium at $t = 1$:

If s_{11} is realized at $t = 1$, the market price recovers its fair value, $V = 1$ and her wealth increases to 0.0538 with 7.6% gain. She is away from the market because the expected return at $t = 2$ conditional upon s_{11} is negative. In contrast, if s_{12} is realized, the market price ($=\psi_{12}$) drops to 0.8313; the realized return of the security is -11.47%. However, she loses more than that due to two driving forces: the leverage itself and the additional funding cost. Altogether her loss is 15.4% and her wealth declines to 0.0423. At this state, she employes leverage upto 83.13% to monetize the enlarged undervaluation.

Equilibrium at $t = 2$:

At s_{21}, the equilibrium price is at its fair value and the arbitrager leaves the market because of zero expected return. If this state is realized via s_{11}, the arbitrager's wealth ($=W_{21|11}$) is idential to her wealth at s_{11}. In contrast, if the state comes through s_{12}, the arbitrager's wealth ($=W_{21|12}$) has increased from 0.0423 to 0.0552 .

(Tranquil State: s_{22}) At s_{22}, $S_{ti} = S$ and therefore the amount of negative noise shock at this state is identical to s_{12}. The magnitude of shock is mediocre and could happen before at $t = 1$ so we define this state as a 'tranquil' state. If this state is realized via s_{11}, her wealth ($=W_{22|11}$) is still 0.0583. The market price ($=P_{22|11}$) is 0.8509 and she employs a leverage ratio of 87.63% ($=\psi_{22|11}$). In contrast, if this state is arrived at through s_{12}, her wealth is reduced to 0.0384; the market price ($=P_{22|12}$) is 0.8282 and her leverage ratio is 1.0375. Given that the amount of negative noise shock is the same across s_{12} and s_{21}, the equilibrium prices are comparable: 0.8313 ($=P_{12}$), 0.8509 ($=P_{22|11}$) and 0.8282 ($=P_{22|12}$). Among the three, $P_{22|11}$ is the highest for obvious reasons; the arbitrager's wealth ($=W_{22|11}$) is the highest and also she will make a sure gain at $t = 3$. In contrast, it is not clear which one should be higher between P_{12} and $P_{22|12}$; W_{12} is higher than $W_{22|12}$, but the expected return at s_{12} is lower than that at s_{22} via s_{12} (in the absence of an arbitrage). The result shows that P_{12} is greater than $P_{22|12}$ and thus the effect of higher wealth dominates the effect of the lower expected return in this case.

(Crash State:s_{23}) We call s_{23} as a 'crash' state given its massive amount of negative shock ($2S = 0.5$) coupled with an extremely low probability of occurrence. In addition, this amount of negative noise shock is unprecedented so it could be counted as an exceptionally rare event. At s_{23} via s_{11}, the arbitrager's wealth ($= W_{23|11}$) is 0.0538 again. The equilibrium price ($=P_{23|11}$) is 0.6802 and the arbitrager steps up the leverage ratio to 2.3511 ($= \psi_{23|11}$). In contrast, at the same state via s_{12}, the arbitrager becomes penurious; her wealth ($=W_{23|12}$) is merely 0.0121, which means 75.8% of her original wealth is wiped out! As such, despite her extensive leverage ratio ($\psi_{23|12} = 3.9318$), the equilibrium price ($=P_{23|12}$) is as low as 0.5598.

The results deliver two essential implications. First, despite the arbitrager's aggressive leverage, the price is not boosted much; the equilibrium price is only 0.5598 whereas its value without arbitrage transaction is $V - 2S = 0.5$. The arbitrager's wealth is extremely low and consequently she does not afford to make a large amount of investment in the asset. Specifically, her total amount of investment is as small as 0.0598. This is substantially lower than 0.0782 at s_{22} via s_{11} despite the fact that the arbitrage opportunity is much more favorable at s_{23}. Note that the crash itself is a double-edged sword. On one hand, the asset price plunges so that the arbitrager's wealth is precipitated. On the other hand, the setback in the asset price delivers an extraordinary opportunity for arbitrage. However, the funding cost structure limits her leverage capacity; when outside funding is most needed, the funding cost is most binding.

Secondly, the impact of the past history on the equilibrium price is quite different between s_{22} and s_{23}. As mentioned above, at the tranquil state, s_{22}, the equilibrium prices are 0.8509 ($=P_{22|11}$) and 0.8282 ($=P_{22|12}$), which are very similar. In contrast, at the crash state, they are 0.6802 ($=P_{23|11}$) and 0.5598 ($=P_{23|12}$), which are quite different from each other. As such, we can state that the price divergence (or volatility) of the equilibrium price is much larger in the crash state than the tranquil state.

Result 1 *The equilibrium price is more volatile during a crash than during a tranquil market period.*

Equilibrium at $t = 3$:

By construction, the price reverts to its fair value, V, at $t = 3$. Among the arbitrager's terminal wealth across different time-paths, $W_{3|23|11}$ is the highest. The arbitrager made a gain from $t = 0$ to s_{11} and then another gain from s_{23} to $t = 3$. She has never lost. In contrast, $W_{3|23|12}$ is the lowest and is even below her initial wealth. She lost twice in a row, from $t = 0$ to s_{12} and from s_{12} to s_{23}. She made a positive gain at s_{23} to $t = 3$, but not enough to recoup her prior losses. It is driven primarily by the progressive funding cost structure; however, the arbitrager's failure to strategically manage the amount of borrowing across time and state exacerbates the loss. The dollar amount borrowed are 0.0140 ($=W_0\psi_0$) at $t = 0$, 0.0390 ($=W_{12}\psi_{12}$) at $s12$ and 0.0477 ($=W_{23|12}\ \psi_{23|12}$) at s_{23}. Remember that $S_0 = 0.125$, $S_{12} = 0.25$ and $S_{23} = 0.5$. Thus when the amount of noise shock doubled from 0.125 to 0.25, the arbirager jacked up the amount of borrowing by 2.79 ($= 0.0390/0.0140$) times. In contrast, when the noise shock doubled from 0.25 to 0.5, the arbitrager increased the amount of borrowing by merely 1.22 ($= 0.0477/0.0390$) times. At s_12, she consumed most of her leverage capacity with expectation that the price would revert to the fair value in the next period. As a result, when the nature bestowed a 'better' opportunity at s_{23}, she ran out of fuel to accelerate the leverage.

2.3.2 The Strategic Arbitrager Model

Figure 2 (b) illustrates the equilibrium of the strategic arbitrager model. Overall the results are more or less similar to those of the schizophrenic arbitrager model. However, consistent with Proposition 5, the strategic arbitrager employs lower leverage across all states by taking account of the impact of his investment on the market price. As a result, P_0 is lower and so are P_{12}, $P_{22|11}$, $P_{23|11}$. In contrast, due to the less aggressive leverage taken, particularly, at s_{12}, her wealth at the crash state is better insulated and thus she affords to substantially leverage her investment, which bolsters $P_{22|12}$ and $P_{23|12}$.

Most importantly, $P_{22|12}$ is higher than $P_{22|11}$ despite the fact that $W_{22|12}$ is lower than $W_{22|11}$. In contrast, in the schizophrenic arbitrager model, $P_{22|12}$ is lower than $P_{22|11}$. This difference highlights the distinguishable feature of the stratetgic arbitrager model. Note that $W_{22|11}$ is fixed at 0.0541. In such a case, the arbitrager is concerned about three effects delivered by $\psi_{22|11}$. An increase in $\psi_{22|11}$ increases the gross dollar return on investment by enlarging the size of total amount of dollar invested. However, it lowers the expected return of the asset by pushing up the price. It also increases the funding cost.

In contrast, when she chooses $\psi_{22|12}$, $W_{22|12}$ is not fixed any more. It is an increasing function of $\psi_{22|12}$ itself; an increase in $\psi_{22|12}$ raises the price, $P_{22|12}$, which, in turn, increases $W_{22|12}$. Such an incrase in wealth reduces the amount of leverage, which results in a lower financing cost. In other words, she can save the financing cost by bulking up her own wealth (by increasing $\psi_{22|12}$). She strategically takes into account this additional *positive* effect that the increase in $\psi_{22|12}$ brings on top of the aforementioned three effects. As a result, *cetris paribus*, she uses more leverage and the equilibrium price becomes higher. For comparision, if $W_{22|12}$ were fixed at 0.0431 (as opposed to varying with $\psi_{22|12}$), her optimal leverage ratio would be substantially lower, 0.5935 and the corresponding equilibrium price would be 0.8188, which is lower than $P_{22|11}$.

Another interesting thing is the arbitrager's terminal wealth. The strategic arbitrager's expected terminal wealth is 0.0562, which is greater than that of the schizophrenic arbitrager, 0.0554. Of course, this is not a surprising result. In addition, the strategic aribtrager's wealth is higher across all states and all paths; the strategic arbitrager does not sacrifice a certain state to increase the expected return. The most noticeable difference between the two models is $W_{23|12}$ and $W_{3|23|12}$. The starategic arbitrager is more conservative at $t = 0$ and s_{12}. The dollar amount of borrowing at $t = 0$ is 0.0019 (=$W_0\psi_0$), which is much smaller than 0.0140 of the schizophrenic arbitrager. She also borrows only 0.0299 (=$W_{12}\psi_{12}$) at s_{12} as opposed to 0.0390 in the schizophrenic arbitrager model. Such conservative moves before the crash arms the strategic arbitrager loaded with more bullets so that she can borrow 0.0721 (=$W_{23|12}$ $\psi_{23|12}$), which is far greater than 0.0476 in the schizophrenic arbitrager model. As a result, her wealth $W_{3|23|12} = 0.0614$ is not only much greater than that of the schizoprehnic arbitrager but also greater than her initial wealth.

2.4 Comparative Statics: q

In this section, we focus on the impact of q on the equilibrium. Note that q, the probability of negative noise shock, determines the strength of mean reversion. At eatch time $t = 1$ and $t = 2$, the probability that the price reverts to its fundamental value, V, is $(1 - q)$. As such, lower q means a stronger mean reversion. If $q = 0$, the market is noise-free thereby mispricing-free as well; Thus, we can say that the market is overall more efficient with lower q.

Figure 3 illustrates how the optimal leverage ratios vary in response to a change in q. Most of them are decreasing with q. With the higher q, the weaker mean reversion (and the higher chance of loss) makes the arbitrager less active in her arbitrage. This is true not only for the schizophrenic arbitrager but also for the strategic arbitrager. As an exception, $\psi_{22|11}$ and $\psi_{23|11}$ are not sensitive to q. The higher q makes the arbitrager more defensive in deciding ψ_0, but for that reason, P_0 is low, which increases its *realized* return at s_{11}. As such, W_{11} is almost the same across q, albeit slightly increasing.[7] Since she does not take any arbitrager position at s_{11}, $W_{22|11}$ and $W_{23|11}$ are identical to W_{11}. As a result, $W_{22|11}$ and $W_{23|11}$ are almost identical with respect to q so that the optimal leverage ratio (and the equilibrium price, $P_{22|11}$ and $P_{23|11}$ displayed in Figure 4) do not vary with q.[8]

Figure 4 shows the impact of q on the equilibrium prices. Except $P_{23|12}$, the equilibrum prices decrease with q or do not respond to q, which reflects the response of the optimal leverage ratios to q. In contrast, $P_{23|12}$ is an increasing function of q. And its sensitivity to q is the strongest among all the equilibrium prices. For example, $P_{23|12}$ is 0.5489 and 0.5885 for $q = 0.01$ and 0.2 respectively. The difference is as large as 7.2%. The arbitrager uses more aggressive leverage at s_{12} with lower q with higher expectation of mean reversion, which results in more loss at s_{23}. As a result, $W_{23|12}$ is much lower and the arbitrge transaction requires a larger amount of leverage, but its funding becomes enormously expensive so the leverage itself is hindered. This results in the lower price with lower q. Therefore, we can conclude that a more efficient market q is more vulnerable to a crash (lower $P_{23|12}$).

Result 2 *A seemingly more efficient market is more vulnerable to a crash.*

Figure 4 also displays the relationship between q and differences in the equilibrium prices at state s_{22} and s_{23}. At s_{22}, the difference between $P_{22|11}$ and $P_{22|12}$ decreases with q but its sensitivity is small; its value is 0.0242 with $q = 0.01$ and 0.0178 with $q = 0.20$, so it decreases by 0.0063. In contrast, the difference between $P_{23|11}$ and $P_{23|12}$ also decreases with q but its sensitivity is much higher; its value is 0.1310 with $q = 0.01$ and 0.0922 with $q = 0.20$, so it falls by 0.0378. Thus we can conclude that price variation (volatility) during the crash (s_{23}) is larger than that during the tranquil time (s_{22}) as documented in Result 1 and, more importantly, such a tendency is more pronounced in a market with lower q.

Result 3 *A seemingly more efficient market shows more extreme tail volatility and a larger difference between tail volatility and non-tail volatlity.*

When we compare the locus of the equilibrium prices between the shizophrenic arbitrager and the strategic arbitrager, the overall results are robust, albeit slightly weaker. For

[7]For example, W_{11} is 0.0537 and 0.0540 for $q = 0.01$ and $q = 0.20$ respectively.

[8]For example, $\psi_{22|11}$ ($\psi_{23|11}$) is 0.8771 and 0.8738 (2.3533 and 2.3451) for $q = 0.01$ and 0.20 respectively.

example, $P_{23|12}$ increases by 0.0178 as q increases from 0.01 to 0.20. So the amount of increase in $P_{23|12}$ is about 47% of that of the schizophrenic arbitrager model. However, such a sensitivity is still much larger than the amount of increase in $P_{22|12}$, which is as small as 0.0015. So the main results documented in Result 2 and Result 3 are still robust.

3 Empirical Analysis

In this section, we empirically test the major implications of our model using the U.S. swap data. Our model delivers three major testable implications, which are documented in Result 1, Result 2 and Result 3. Out of the three implications, we focus on Result 2 and Result 3. Result 1 is not straightforward to test because we need to identify the whole state space of S similar to the one in Figure 1. The state space of S itself may be equipped with a larger volatility at its tail. Then the higher tail volatility of the market price could be driven by that rather than the strong path-dependency of the equilibrium price at the crash.

We use the fixed-income market as a natural candidate for testing our model. Fixed-income arbitrage is one of most popular strategies employed by hedge funds. As its name implies, it is an investment strategy that attempts to exploit mispricing which develops among related classes of fixed-income securities. Strictly speaking, it is a statistical arbitrage since mispricing is identified by a statistical analysis rather than by a strict economic reasoning. A representative strategy is to exploit a substantial deviation of a particular spread (such as yield spread, basis between cash and futures, credit spread) from its historical average. To eliminate or minimize its exposure to a fundamental risk, this strategy takes long positions in one asset and short positions in another asset(s). For that reason, the strategy is called 'relative value trading strategy.'[9]

This strategy is generically designed to eliminate its exposure to market risk and credit risk; as such, the expected return on a dollar investment is relatively small so an unusually high degreee of leverage is inevitable and often emphasized. In other words, its underlying

[9] A Long/short equity strategy is another popular strategy utilized by hedge funds. It also simultaneously takes long and short positions in equity space. However, most of them are long biased (such as 130/30, where long exposure is 130% and short exposure is 30%) and it is composed of a 'long' portfolio by buying equities that are expected to increase in value and a separate 'short' portfolio by shorting equities that are expected to decrease in value. Sinice it does not eliminate systematic risk completely, it is called a 'fundamental' long/short strategy. There is a pairwise long/short equity strategy, which matches a particular stock that it is long (short) on to another stock with a similar risk profile such as beta. This strategy is not actively adopted by hedge funds though because remaining idiosyncratic risk after controlling the market risk is still sizable and its compensation is relatively small.

driver is not to take systematic risk while taking leverage risk: a giant vacuum cleaner sweeping up pennies.

There exist a several number of studies which empirically analyze the limited arbitrage. Mitchell and Pulvino (2001) analyze mergers to characterize the risk and return in risk arbitrage and Mitchell, Pulvino and Stafford (2002) investigate situations where the market value of a company is less than its subsidiary. Kapadia and Pu (2009) propose limits to arbitrage as an explanation for a low correlation between equity and credit markets and test it. Mancini-Griffoli and Ronaldo (2011) investigate how covered interest parity broke down in the aftermath of the global financial crisis by focusing on the funding liquidity. Recently, Jermann (2017) shows that negtive swap spread, which implies a risk-free arbitrage opportunity, can be explained by introducing frictions for holding bonds.

Among existing studies on limited arbitrage, the empirical analysis of Duarte, Longstaff and Yu (2006) is most related to our study. They document that the fixed-income arbitrage strategies produce significant alphas after controlling for bond and equity market risk factors and many of them produce positively skewed returns. Thus, they conclude that it is not sensible to derogate the fixed-income arbitrager for 'picking up nickels in front of a steamroller.' However, they investigate the risk and return chracteristics of representative arbitrage trading straegies in the fixed-income sector that they construct, **not** the actual returns of fixed-income arbitrage funds.[10] In addition, their data period ends before the outbreak of the subprime mortgage crisis. Thus, their conclusion might be premature because they did not have a chance to see the genuine steamroller.

To see what happened in fixed-income arbitrage funds during the global financial crisis, we investigate a monthly average returns of fixed-income arbitrage funds from 1997 to April of 2017, which are provided by the Barclay Fixed Income Arbitrage Index. Figure 5(a) depcits the time series evoloution of annual returns of the fixed-income arbitrage funds. Despite the demise of LTCM, the cross-sectional average return in 1998 is still positive, 0.76%. However, the global financial crsisis was a much more catarostrophic havoc to the industry of fixed-income arbitrage funds. They lost, on average, 0.60% in 2007, for the first time in their history. In the following year, their return nosedived to -25.20%! Before 2007 that Duarte, Longstaff and Yu's data covers, the hedge funds' average monthly return is as high as 66 basis point. Its standard deviation is as small as 87 basis point. Its skewness is -2.08, slightly negative, but not statistically significant. Its excess kurtosis is 9.70.[11] If

[10]They investigate swap spread arbitrage, yield curve arbitrage (not slope/butterfly spread strategies though), mortgage arbitrage and fixed-income volatility arbitrage.

[11]A number of trading strategies examined in Durate, Longstaff and Yu produce positvely skewed returns. In contrast, the actual returns of the fixed income arbitrage funds are negatively skewed as shown in Figure 5(b). This is true even when we use only the data before the global financial crisis. A survivorship bias

adding the post-crisis data till Aprial of 2017, the above-mentioned figures become quite different. Its mean drops to 49 basis point; its standard deviation jumps up to 141 basis point; the return becomes more negatively skewed, -4.87, and more leptokurtic with excess kurtosis of 42.11. Figure 5(b) shows that the distribution of monthly returns is extremely negatively skewed. That is, the hidden dark side of the fixed income arbitrage emerged with the outbreak of the global financial crisis!

Existing studies including the aforementioned studies have not investigated a relationship between a mean-reversion speed (or a strength of convergence) and a tail behavior of arbtirage payoffs (Result 2 and Result 3). Mitchell, Pulvino and Stafford (2002) show that returns to an arbitrager would be 50% larger if the path to convergence was smooth rather than as observed in the data. In our model's context, the arbitrager's ex-post return would be higher (setting aside the higher expected return) if the path is less volatile because the arbitrager can save the funding cost given the progressive fuding cost structure. However, that result is not directly related to what we want to empirically analyze, which are Result 2 and Result 3.

We employ the U.S. interest rate swap market as a natural candidate for testing the theoretical implications. The interest rate swap market is one of most preferred habitat for hedge funds and proprietary desks of global investment banks. Trading strategies involving interest swaps encompass not only generic yield spreads such as slopes and butterflies of a particular swap curve, but also asset swap margins combined with cash bonds, cross-country basis swaps and basis on futures and their combinations. Given that there are so many different strategies available, most of the tradig desks employ a very sophisticated quantitative algorithm called 'trade finder' to detect best opportunities available in a real time basis. Herein we focus on the most simple and classic trading strategies surrounding the interest rate swaps, slope spreads and butterflies.

3.1 Trading Strategies Using the Swap Yield Curve

Herein we investigate the most popularly used yield curve strategies among the fixed-income arbitrage funds: slope and butterfly spreads.

(1) Slope Spreads

The slope strategies center upon a yield difference between a longer-end and a shorter-end of a yield curve. For example, '2s10s' is a generic industry jargon for a spread between

cannot be an explanation for this discrepancy. Such discrepancy may result from the fact that there are many other strategies not considered by Durarte, Longstaff and Yu and they produce significantly negatively skewed returns. A further study is needed to clarify this.

the ten-year yield and the two-year yield. There exist many different strategies using a particular slope spread, and the most popular one is the so-called 'duration matched' strategy. For a short trading horizon, a profit from a fixed leg in the long end swap can be approximated by

$$\Delta V_l \approx -D_l V_l \Delta y_l,$$

where D_l is the duration of the fixed leg. Thus a trading position with long on the long-end and short on the shorter-end with a ratio of $1 : \lambda_s$ is

$$\Delta(V_l - \lambda_s V_s) = \Delta V_l - \lambda_s \Delta V_s \approx -D_l V_l \Delta y_l + \lambda_s D_s V_s \Delta y_s.$$

That is, the trading position's short-term payoff is described as a linear function of the two yields. We want to eliminate the position's exposure to the parallel shift of the yield curve, a directional market risk. Thus we determine λ_s such that

$$-D_l V_l + \lambda_s D_s V_s = 0 \quad \Longrightarrow \quad \lambda_s = \frac{D_l V_l}{D_s V_s} = \frac{D_l}{D_s}.$$

The last equality is based on an assumption that both swaps are par-par swaps. A par-par swap is a generic swap which designates the par notional amount to a fixed leg as well as a floating leg. As such, $V_l = V_s$.[12] Putting this value of λ_s back into the payoff of the position yields

$$\Delta(V_l - \lambda_s V_s) \approx \left[-D_l \Delta y_l + \frac{D_l}{D_s} D_s \Delta y_s\right] V_l = -D_l V_l \;\; \Delta(y_l - y_s).$$

Thus, the profit of the aforementioned trading stragy is approximately proportional to the change in the yield spread, $y_l - y_s$. If the yield curve shifts parallelly (i.e., $y_l - y_s = 0$), the arbitrage earns zero. If the curve steepens (i.e., $y_l - y_s > 0$), the strategy loses and vice versa. In the industry, $y_l - y_s$ is called 'pick-up,' and the fund managers meticulousely monitor its change on a real time basis. Typically, they compute the z-value of the spread based on the past six-month or one-year history and when the z-value is greater or less than a particular threshold level, they consider entering into a position. For example, if the six-month z value is greater than 2.0 (less than -2.0), they believe tha the curve is abnormally steep (flat) so the fund managers take a position in a flattener (steepener): i.e., receive (pay) the longer-end and pay (receive) the shorter-end. Because the strategy's market exposure (duration risk) is eliminated, its expected profit is quite small and therefore the fund has to use a very high degree of leverage, which could sometimes be as high as ten or twenty times.

[12]There is non par-par swap, which is tailored to a specific need of a client; the notional value of each leg is typically not a par value due to a stub period.

There are many variations of the above-mentioned trading strategy. Most of them are based on a belief that the directional move of the curve does not acompany a parallel shift but a certain change in a shape of the curve. For example, when the market interest rate rises in the absence of a monetary policy, the slope upto its belly part tends to steepen while the long-end slope flattens (short-end bear steepening and long-end bear flattening). When the market interest rate falls, the opposite is more likely to occur: short-end bull flattening and long-end bull steepening. Thus, to capture such a statistical relationship between the direction of the yield curve and the correspondig change in the slopes, the quant managers use regression, Principal Component Analysis (PCA) or Independent Component Analysis (ICA) to adjust the ratio of the long-end and the short-end. However, the most popular and representative type of a slope trading strategy is the above-mentioned one.

(2) Butterfly Spreads

A butterfly strategy involves three tenors as opposed to two tenors. For example, '5s10s20s' refers to a spread between the ten-year yield (middle leg) and the average of the five-year yield (short leg) and the twenty-year yield (log leg). Thus it measures the curvature of the swap yield curve. If the curve is expected to be more more concave, the trader 'pays' the butterfly (short on the middle leg and long on the combination of the short and the log legs) and vice versa. Thus the butterfly is associated with the third factor of the PCA, e.g., the curvature factor whereas the slope is related with the second factor.[13]

Below, we investigte the most widely used strategy, a double duration mathched butterfly. The value change of the butterfly can be again approximated as

$$-D_m V_m \Delta y_m + \lambda_s D_s V_s \Delta y_s + \lambda_l D_l V_\Delta y_l.$$

We determine λ_s and λ_l such that (i) the duration of the butterfly is zero and (ii) the duration of the short leg is identical to the duration of the long leg:

$$\begin{aligned} -D_m V_m + \lambda_s D_s V_s + \lambda_l D_l V_l &= 0 \\ \lambda_s D_s V_s - \lambda_l D_l V_l &= 0, \end{aligned}$$

which yields

$$\lambda_s = \frac{D_m V_m}{2 D_s V_s} = \frac{D_m}{2 D_s} \text{ and } \lambda_l = \frac{D_m V_m}{2 D_l V_l} = \frac{D_m}{2 D_l}.$$

Again we assume that the swaps are the par-par swaps in the last equalities in the above two equations. Then the payoff can be described as

$$-D_m V_m \Delta y_m + \lambda_s D_s V_s \Delta y_s + \lambda_l D_l V_\Delta y_l = -D_m V_m \Delta \left[y_m - \frac{1}{2}(y_s + y_l) \right].$$

[13] A simple 'pay' or 'receive' of a particular tenor, which is equivalent to shorting or longing a cash bond is a directional bet on the yield and thus is related to the first factor of PCA.

Thus the payoff of the butterfly is proportional to the difference between the yield of the middle leg and the average yield of the short leg and the long leg. If the trader 'receives' the (middle leg of the) butterfly, she gains if the curve becomes less curvatured and loses if the opposite happens. Similar to the slope spreads, there are many other variations which utilize the statistical association among the three legs such as a regression analysis, a PCA and an ICA.

One thing to mention is the funding cost associated with spread strategies. A swap contract does not accompany any exchange of notional values because the notional value of a fixed leg is identical to that of a floating leg. And its funding cost is already embedded within its contract; if you receive the fixed leg, its funding rate is the six-month LIBOR rate, a coupon rate of the floating leg. If you receive the floating leg, its funding rate is the swap rate. As such, even if the size of swap position (notional value) increases, the funding rate itself does not increase. The same argument can be made about a spread position, a combination of swaps with different tenors. However this logic does not reflect the market practice of marking-to-market (MTM) and collateralization. Under the MTM practice, counterparties are required to post collateral in the amount of the mark-to-market value of the contract.[14] When the mark-to-market value of one party in a swap contract is negative, she needs to pay collateral to her counterparty in the amount of loss. In that sense, it is similar to a futures contract as opposed to a forward contract.[15] Collateral is costly to post, so it induces economic costs to the collateral payer. If she continues to lose in the mark-to-market value of her position, she needs to post additional collateral and this cost rises concommitantly. If she fails to post it, she becomes bankrupt. Most of the collateral posted is in the form of cash or Treasury securities. To pay the collateral, she may use her own cash or Treasuries; otherwise she needs to finance it. Since she loses more, she needs to finance it more and hence the financing cost may rise as well.[16] Therefore, *a priori*, a swap position or its combinations (including spreads) entails an implicit funding cost; this cost tends to increases with the size of potential loss in the position. In turn, the size of potential loss is proportional to the size of position and also the riskiness of the position. Overall, this feature is in line with Assumption 1 in our model that the funding rate is proportional to the leverage ratio.

[14] See Johannes and Sundaresan (2007) for this market convention and its impact on the swap rates.
[15] See Johannes and Sundaren (2007) for details.
[16] Or her counterparty, typicall a dealer, applies higher haircuts to Treasuries collateralized.

3.2 Data

We use the U.S. swap yield data from July 23, 1998 to May 11, 2017 from the Bloomberg. We eliminate weekends and holidays. The number of daily observations is 4904. The corresponding number of weekly observations (Wendesday) is 980.[17] The tenors are from one year to ten year along with fifteen year, twenty year and thirty year and thus the total number of tenors is thirteen. Consequently the number of slope spreads and butterfly spreads we can construct is 78 ($=\sum_{i=1}^{12} i$) and 286 ($=\sum_{i=2}^{12}(i-1)(13-i)$) respectively.

3.3 Tests

Among many implications delivered by our model, we focus on testing two major hypotheses.

(H1): The spreads with stronger mean reversion are subject to higher tail risk.

(H2): The spreads with stronger mean reversion are subject to higher tail volatility risk.

Going back to the model, (H1) corresponds to Result 2, which is grounded on the fact that $P_{23|12}$ increases with q. (H2) is built upon Result 3, which reflects the fact that a discrepancy between $P_{23|11}$ and $P_{23|12}$ decreases with q.

Both hypotheses require the operational definition of 'tail' states (corresponding to s_{23} in our model). We consider 0.5 percentile, 1 percentile and 2 percentile as threshold levels. Even though we theoretically investigate only when the asset is undervalued due to negative noise shocks, a symmetric result holds when the asset is overvalued due to positive noise shocks, as long as the funding cost of shorting increases with the size of short position. Thus we investigate both tails of a spread distribution.

To test the hypotheses, we first normalize the spreads to their corresponding z values. By doing so, we can equalize the risk amount of trade across spreads and thus the scale of potential collateral. Therefore we can directly compare their tail risk and tail volatility risk cross-sectionally.[18] The test procedure is composed of two steps. In the first step, we estimate the mean-reversion speed of each spread from its time-series data. We also

[17] We use other weedays to check the robustness of our results. Qualitatively speaking, the results are the same.

[18] For example, the payoff of a slope spread is approximately $-D_l V_l \Delta(y_l - y_s)$. After normalizing $y_l - y_s$, the only remaining difference across different slopes is D_l. As long as the duration does not change much over time, D_l is close to constant over time. In such a case, normalizing $D_l(y_l - y_s)$ is quanlitatively

estimate the statistics associated with tail risk and tail volatility risk. In the second step, we run a cross-sectional regression on those statistics against the mean-reversion speed.

(1) First step: time series estimation
We estimate each spread's mean-reversion speed, δ_i by

$$\Delta z_{i,t+1} = \mu_i + \delta_i z_{i,t} + \epsilon_{i,t+1},$$

where $z_{i,t}$ is a normalized spread i at time t. The mean-reversion speed is $-\delta_i$.

In testing (H1), we adopt the following three alternative measures for tail risk:

(i) kurtosis $\left(= \frac{\hat{E}(z_{i,t})^4}{\hat{\sigma}(sp_i)^2}\right)$.

(ii) p-percentile Value at Risk (VaR): $z_{i|p}$ for the left tail, $z_{i|1-p}$ for the right tail.

(iii) Short Fall Risk: $\hat{E}(z_{i,t}|z_{i,t} < z_{i|p})$ for the left tail, $\hat{E}(z_{i,t}|z_{i,t} > z_{i|1-p})$ for the right tail

VaR and Short Fall Risk are estimated by historical simulation. Each measure has its own strength and weakness and we do not want to discuss them in detail. Simply we apply all these measures together.

The measure of tail volatility needed in testing (H2) is a standard deviation conditional upon the occurrence of a spread beyond the threshold value mentioned above:

$$\sigma_{i|p-} = \sigma\left(z_{i,t}|z_{i,t} < z_{i,t|p}\right) \quad \sigma_{i|p+} = \sigma\left(z_{i,t}|z_{i,t} > z_{i,t|(1-p)}\right),$$

where $p = 0.005$, 0.01 or 0.02.

(2) Second Stage: Cross-sectional Regression
In the second stage, we run the following regressions:

$$y_i = \beta_0 + \beta_1 \delta_i + e_i.$$

where y_i is the kurtosis, VaR, short fall risk and tail volatility of spread i. For comparison, we also run a regression on non-tail volatility of spreads.

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equivalent to normalizing $(y_l - y_s)$ since

$$\frac{D_l(y_{lt} - y_{st}) - \mu(D_l(y_{lt} - y_{st})}{D_l\sigma(y_{lt} - y_{st})} = \frac{y_{lt} - y_{st} - \mu(y_{lt} - y_{st})}{\sigma(y_{lt} - y_{st})}.$$

Simply put, taking $\frac{V_l}{\sigma(y_{lt} - y_{st})}$ as a notional value of the position assigns the same risk profile to different spreads.

However, this two-step estimation procedure suffers from a classic errors-in variables (EIV) problem. We use the estimate of mean-reversion speed as an independent variable and the estimates of distributional characteristics (kurtosis, Value-at-Risk, expected short-fall risk and etc.) as dependent variables. Therefore, both regressors and regressands are subject to estimation errors. As is well known, though, the measurement errors in regressands do not cause a problem as long as they are uncorrelated with the regressors and their measurement errors.

Thus we focus on the errors-in-variable of the regressors. The EIV leads to a bias in the estimated coefficients, toward zero, which is called 'attenuation bias.' In our empirical work, we find that the corrections for this bias are, in general, relatively small. Specific methods to adjust the EIV problem are presented in Appendix C.

3.4 Estimation Results

Table 1A Table 1C report the estimation results for slopes, butterflies and all of them respectively. In Table 1A, kurtosis is positively associated with mean-reversion speed with statistical significance and R^2 is as high as 59.9%. Next, the mean-reversion speeds demonstrate a negtive relationship with negative VaRs and a positive relationship with positive VaRs. That is, the slopes with stronger mean reversion is more likely to have a fat left tail risk as well as a right tail risk. We find the similar findings, albeit stronger when employing shortfall risk as an alternative measure of tail risk. In addition, such a result is more pronounced when the threshold percentile, p, is smaller, which means that the more serious tail risk, it is more strongly associated with the mean-reversion speed. All these results hold when we use weekly data as well. Overall we can conclude that hypothesis (H1) is strongly supported.

In addition, both tail volatilities, left tail and right tail, are also strongly positvely associated with mean-reversion speed. In contrast, non-tail volatilities are not significantly associated with mean reversion speed. The results are robust to when we use different threshold levels and weekly data. So we can conclude that hypothesis (H2) is also well supported.

Table 1B reports the estimation results for the butterfly spreads. Qualitatively speaking, the overall results are similar to Table 1A. The VaR results are less strong but the results with shortfall risk confirm that the tail risk is higher with stornger mean-reverting butterflies.[19] Another thing to notice is that non-tail volatilities are negatively associated,

[19]The estimation of VaR is, in general, less efficient since it estimates one particular point. In contrast, the shortfall risk estimates the 'mean' of sample observations below (above) that negative (positive) VaR

rather than positively associated, with mean-reversion speed. That is, the butterflies with stronger mean-reversion speed tend to have lower volatilities during tranquil time. Table 1C reports the results when the slopes and the butterflies are all adopted as dependent variables. The overall results are again similar to Table 1A and 1B. Both hypotheses are well supported.

Table 2A, 2B and 2C are based on EIV-adjustments. As expected, the findings in Table 1A, 1B and 1C become stronger by alleviating the attenuation biases arising from EIVs.

Finally, we reexamine the relationship by using the mean-reversion speed during tranquil time only. We are concerned about the possibility that the estimates of mean-reversion speed are contaminated. Specifically, the mean-reversion might be accelerated during a crash and such a tendency may be more pronounced with spreads which suffer more during a period of market stress. Thus we re-estimate the mean-reversion speed of each spread by excluding the tail part. The estmation results are reported in Table 3A, 3B and 3C. The overall results are still robust to these alternative measures of mean-reversion speeeds. The only noticeable difference is that, in Table 3A, the positive non-tail volatilties of slopes are positively associated with the mean-reversion speeds with statistically significance. However, the regression coefficients are still well below those of positive tail volatilities. Thus (H2) is still supported.

In summary, we can conclude that the two hypotheses, (H1) and (H2), are well in line with the behavior of the U.S. swap curve. A seemingly more efficient market is more likely to be dismantled and also is subject to higher volatility once the crash occurs.

4 Conclusion

This paper delivers a novel insight on limited arbitrage on top of exisiting literature by centering upon what kind of market is more likely to attract arbitrage transactions and demonstrating theoretically and empirically that such a market is more susceptible to a crash.

Hedge funds specializing in fixed income arbitrage are extremely similar in their key strategies. They seize a trade opportunity when the gap between the market price of a security and its fair value widens above a pre-specified level. They unwind their positions either when the spread contracts to a certain level (profit realization). Therefore, their entry into and exit from trades are very similar albeit their exact profit realization levels and loss cut levels being slightly different. Simply put, their investment strategies are uni-

level.

directional and from the lack of diversity. As a result, regardless of the number of hedge funds participating in the JF market, they act as a huddled mass.[20]

Under a tranquil market condition, such synchronized collective actions among hedge funds have the benefits of polishing the market more effectively by eliminating mispricings quickly and sufficiently. However, when the market is embroiled in turmoil, i.e., when it is time that the arbitrager's demand is most needed, the arbitrage mechanism itself malfunctions and fails to correct dislocations in prices. In our model, an arbitrager is ensured to survive until the market price converges to its fair value. In addition, we do not introduce a loss-cut practice that is widely employed by hedge funds.[21] As such, in our model, the arbitrager exits the market only if she earns gains and does not expect any further profit opportunity. If we allow other reasons including the aforementioned ones the arbitrager leaves the market (so when the gap widens rather than shrinks), the model may amplify the mispricing; for example, $P_{23|12}$ could be even lower than $V - 2S$. In the worst case, the market collapses and fails to be resurrected as evidenced by Japanese floating rate bond market.[22] We reserve this kind of extension for future research.

[20]Thus our assumption of a single arbitrager is not entirely preposterious.

[21]See Ahn, Kim and Seo (2017) for a potential equilibrium of disequilibrium in the presence of such a practice.

[22]See Ahn, Baek, Chung and Kang (2016) for how the Japanese floater market collapsed in the aftermath of the global financial crisis.

Appendix A: Proofs of Propositions

Proof of Proposition 1:

(i) When $\frac{E(P_{t+1}|s_t)}{P_{s_t}} > 1$: the first order condition yields

$$\psi^*_{s_t} = \frac{\frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1}{2\phi} > 0.$$

(ii) When $\frac{E(P_{t+1}|s_t)}{P_{s_t}} = 1$: a condition for $E[W_{t+1}|W_{s_t}] > W_{s_t}$ becomes

$$-\phi\psi^2_{s_t} 1_{\psi_{s_t}>0} > 0.$$

Therefore, $\psi_{s_t} \leq 0$, thereby making $1_{\psi_{s_t}>0} = 0$, is optimal; the arbitrager does not employ any leverage. She is indifferent between riskless saving and arbitrage transaction with her own wealth or their combinations.

(iii) When $\frac{E(P_{t+1}|s_t)}{P_{s_t}} < 1$:

(a) If the arbitrager chooses positive ψ_{s_t}, a condition for $E[W_{t+1}|W_{s_t}] > W_{s_t}$ is

$$\psi_{s_t}\left[\frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1\right] + \frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1 - \phi\psi^2_{s_t} > 0.$$

It is a quadratic function which achieves its maximum at $\psi_{s_t} = \frac{\frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1}{2\phi} < 0$. As such, for a positive domain of ψ_{s_t}, the function is downward sloping with an intercept of $\frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1$. Consequently, the supremum over $\psi_{s_t} \geq 0$ is the intercept itself, $\frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1$, which is negative.

(b) If the arbitrager chooses negative ψ_{s_t}, the condition becomes

$$\psi_{s_t}\left[\frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1\right] + \frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1 > 0.$$

The expression in the left hand side is a downward linear function of ψ_{s_t} with an intercept, $\frac{E(P_{t+1}|s_t)}{P_{s_t}} - 1$.

Combining (a) and (b) indicates that the overall shape of $E[W_{t+1}|W_{s_t}]$ when $\frac{E(P_{t+1}|s_t)}{P_{s_t}} < 1$ is a combination of the downward linear function and the downward quadratic function, which meet each other at $\psi_{s_t} = 0$. Consequently, the supremum occurs $\psi_{s_t} = -1$, the lowest value of leverage ratio.

Lemma 1 *The wealth of the arbitrager at state s_{12} under $\psi_{12} = 0$ is always smaller than the initial wealth; i.e., $(W_{12}|\psi_{12} = 0) < W_0$.*

Proof: Suppose that $\psi_0 = 0$. Then, $P_0 = V - \frac{1}{2}S + W_0$.

$$\begin{aligned}(W_{12}|\psi_0 = 0, \psi_{12} = 0) &= W_0 \frac{(P_{12}|\psi_{12} = 0)}{(P_0|\psi_0 = 0)} \\ &= W_0 \frac{V - S + (W_{12}|\psi_0 = 0, \psi_{12} = 0)}{V - \frac{1}{2}S + W_0}.\end{aligned}$$

Solving for $(W_{12}|\psi_0 = 0, \psi_{12} = 0)$ yields

$$(W_{12}|\psi_0 = 0, \psi_{12} = 0) = W_0 \frac{V - S}{V - \frac{1}{2}S} < W_0. \tag{17}$$

Therefore, when the arbitrager does not take any leverage at time 0, $W_{12} < W_0$. Now we show that if the arbitrager increases her leverage from 0, W_{12} decreases. For brevity, we suppress the conditional argument on $(W_{12}|\psi_{12} = 0)$ and $(P_{12}|\psi_{12} = 0)$ from here on. Note that

$$\begin{aligned}W_{12} &= W_0 \left[(1 + \psi_0)\frac{P_{12}}{P_0} - \psi_0(1 + \phi\psi_0)\right] \\ &= W_0 \left[(1 + \psi_0)\frac{V - S + W_{12}}{V - \frac{1}{2}S + W_0(1 + \psi_0)} - \psi_0(1 + \phi\psi_0)\right].\end{aligned}$$

The solution to W_{12} is

$$W_{12} = \frac{W_0}{V - \frac{1}{2}S}\left[(1 + \psi_0)(V - S) - \left(V - \frac{1}{2}S + W_0(1 + \psi_0)\right)\psi_0(1 + \phi\psi_0)\right]$$

Then its derivative with respect to ψ_0 is

$$\frac{\partial W_{12}}{\partial \psi_0} = \frac{W_0}{V - \frac{1}{2}S}\left[-3\phi W_0 \psi_0^2 - 2\left[W_0 + \phi\left(V - \frac{1}{2}S + W_0\right)\right]\psi_0 - \left(\frac{1}{2}S + W_0\right)\right].$$

The expression inside the square bracket is a quadratic equation and its roots are

$$\psi_0^* = \frac{b \pm \sqrt{b^2 - 12\phi W_0\left(\frac{1}{2}S + W_0\right)}}{6\phi},$$

where $b = -2\left[W_0 + \phi\left(V - \frac{1}{2}S + W_0\right)\right] < 0$. Therefore, both roots are all negative, and thus

$$\frac{\partial W_{12}}{\partial \psi_0} < 0 \quad \forall\ \psi_0 \geq 0.$$

Combining the two facts, $(W_{12}|\psi_0 = 0) < W_0$ and $\frac{\partial W_{12}}{\partial \psi_0} < 0\ \forall\ \psi_0 > 0$ means $W_{12} < W_0$ for all $\psi_0 > 0$. □

Proof of Proposition 2:

We first prove that the arbitrager always takes positive leverage at time 0 if the proposed condition is met. Then we show that the arbitrager always takes leverage afterwards.

(i) $\boldsymbol{t=0}$ **:** Suppose that the arbitrager does not take leverage and simply invests her own initial wealth, W_0 at $t=0$. To justify any epsilion amount of leverage, the expected return on the security should be positive or equivalently the price should be undervalued despite the investment of the arbitrager:

$$(P_0|\psi_0=0)=V-\frac{1}{2}S+W_0<E(P_1)=(1-q)V+qP_{12}.$$

Note that $P_{12}=V-S+W_{12}(1+\psi_{12})$ where $\psi_{12}\geq -1$ as shown below. $E(P_1)$ is endogenously determined in equilibrium because so is P_{12}. However we can still identify a sufficient condition for an admissibility of leverage strategy, $\psi_0>0$. Given the feasible range of ψ_0, a sufficient condition is

$$(P_0||\psi_0=0)=V-\frac{1}{2}S+W_0<\inf_{\psi_{12}}E(P_1)=((1-q)V+q(V-S)),$$

which leads to $S>\frac{2W_0}{1-2q}$. Thus when $S>\frac{2W_0}{1-2q}$, the arbitrager alwyas use positive leverage at $t=0$.

(ii) $\boldsymbol{t=1}$ **:** At $t=1$, we need to focus on s_{12} only since at s_{11}, there is no noise shock and thus the arbitrager will be dormant in this market. At s_{12}, the arbitrager would leverage her investment if

$$P_{12}=V-S+W_{12}(1+\psi_{12})<E(P_2|s_{12})=(1-q)V+\frac{q}{2}(P_{22}|s_{12})+\frac{q}{2}(P_{23}|s_{12}). \quad (18)$$

Again, to validate her positive leverage, we need a sufficient condition that the conditional expected return is still positive when $\psi_{12}=0$ and that is true even at the lowest feasible prices, $P_{22}|s_{12}$ and $P_{23}|s_{12}$. Given the fact that

$$P_{22}|s_{12}=V-S+(W_{22}|s_{12})(1+\psi_{22}),\quad P_{23}|s_{12}=V-2S+(W_{23}|s_{12})(1+\psi_{23}),$$

and thus (18) can be rewritten as:

$$\begin{aligned} V-S+W_{12} &< \inf_{\psi_{22},\psi_{23}}E(P_2|s_{12})=(1-q)V+\frac{q}{2}(V-S)+\frac{q}{2}(V-2S) \\ &= V-\frac{3}{2}qS. \end{aligned}$$

This condition is reduced to

$$S>\frac{2}{2-3q}W_{12},$$

and this is a sufficient condition for $\psi_{12}>0$. It is straightforward to show that

$$\frac{2}{1-2q}>\frac{2}{2-3q}\quad\forall\ 0<q<\frac{1}{2}.$$

Following Lemma 1, $W_0>W_{12}$; Therefore,

$$S>\frac{2}{1-2q}W_0>\frac{2}{1-2q}W_{12}>\frac{2}{2-3q}W_{12}.$$

At $t=2$, the arbitrage always use leverage since the asset recovers its fair value at $t=3$. This completes the proof. □

Proof of Propositin 3:

To find a condition for $W_{23|12} > 0$, we need to find ψ_{12}^* such that

$$\begin{aligned}\psi_{12}^* &= \text{argmax } E(W_{2|12}) = W_{12}\left[(1+\psi_{12})\frac{(1-q)V + \frac{q}{2}P_{22|12} + \frac{q}{2}P_{23|12}}{P_12} - \psi_{12}(1+\phi\psi_{12})\right] \\ &= \frac{\frac{(1-q)V + \frac{q}{2}P_{22|12} + \frac{q}{2}P_{23|12}}{P_{12}} - 1}{2\phi}\end{aligned}$$

Plugging back ψ_{12}^* into $W_{23|12}$ yields the desired condition. However, $P_{22|12}$, $P_{23|12}$ and P_{12} are determined in the equilibrium, and unfortunately they are all numerically solved. Below, we approximate the condition by replacing the market prices by the value in the absence of arbitrage demand. We first solve for a pseudo optimal leverage ratio at s_{12} such that

$$\begin{aligned}\psi_{12}^* \approx \psi_{12}^p &= \text{argmax } E(W_{2|12}^p) = W_12\left[(1+\psi_{12})\frac{(1-q)V + \frac{q}{2}(V-S) + \frac{q}{2}(V-2S)}{P_12} - \psi_{12}(1+\phi\psi_{12})\right] \\ &= \frac{\frac{(1-q)V + \frac{q}{2}(V-S) + \frac{q}{2}(V-2S)}{V-S} - 1}{2\phi} \\ &= \frac{S\left(1-\frac{3}{2}q\right)}{2\phi(V-S)}\end{aligned}$$

We also approximate $W_{23|12}$ as following:

$$\begin{aligned}W_{23|12}^p &= W_{12}\left[(1+\psi_{12})\frac{V-2S}{V-S} - \psi_{12}(1+\phi\psi_{12})\right] \\ &= W_{12}\left[-\phi\psi_{12}^2 - \frac{S}{V-S}\psi_{12} + \frac{V-2S}{V-S}\right].\end{aligned}$$

Note that the expression inside the square bracket is a quadratic function with an inverted U shape. It has two roots: one is negative and the other is positive. We consider only the positive leverage ratio so that the valid root is a positive one. Then, the feasible set of ψ_{12} which yields $W_{23|12}^p > 0$ is

$$\psi_{12} \in \left[0, \frac{\frac{S}{V-S} - \sqrt{\left(\frac{S}{V-S}\right)^2 + 4\phi\frac{V-2S}{V-S}}}{-2\phi}\right).$$

Therefore, for $W_{23|12}$ to be positive, the above approximate solution, ψ_{12}^p should be inside this feasible set or equivalently,

$$\psi_{12}^p = \frac{S\left(1-\frac{3}{2}q\right)}{2\phi(V-S)} < \frac{\frac{S}{V-S} - \sqrt{\left(\frac{S}{V-S}\right)^2 + 4\phi\frac{V-2S}{V-S}}}{-2\phi} \qquad (19)$$

Simplifying the above inequality condition yields

$$S^2\left[\left(2-\frac{3}{2}q\right)^2 - 1 - 8\phi\right] + 12\phi SV - 4\phi V^2 \leq 0. \qquad (20)$$

The two roots are

$$S = \frac{-12\phi V \pm \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} \tag{21}$$

The feasible range of S^p depends on the sign of the numerator of (20).

(i) When $\left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi > 0$.
In this case, (20) implies

$$\frac{-12\phi V - \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} < S < \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}.$$

The value of the left hand side is negative whereas the value of the right hand side is positive. Since we consider only the negative noise shock, $S > 0$. Therefore the approximate upper limit of S is

$$S^p = \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}.$$

(ii) When $\left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi = 0$.
From (20), we can get $S < \frac{V}{3}$ so that

$$S^p = \frac{V}{3}.$$

(iii) When $\left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi < 0$.
In this case,

$$S < \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} \tag{22}$$

$$\text{or } S > \frac{-12\phi V - \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} \tag{23}$$

Below we show that the second inequality is not valid. To see this, we first show that

$$\begin{aligned}\lim_{\phi \uparrow \infty} \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} &= \frac{-12V + 4V}{-16} \\ &= \frac{V}{2}\end{aligned}$$

and

$$\begin{aligned}\lim_{\phi \uparrow \infty} \frac{-12\phi V - \sqrt{(12\phi V)^2 + 16\phi V^2 \left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}}}{2\left\{ \left(2 - \frac{3}{2}q\right)^2 - 1 - 8\phi \right\}} &= \frac{-12V - 4V}{-16} \\ &= V.\end{aligned}$$

Therefore, the maximum value of the left hand side in (20) is obtained at $S = \frac{3}{4}V$. Second, we show that it is also a decreasing function of ϕ. That is,

$$\frac{\partial \frac{-12\phi V}{2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}}}{\partial\phi} = \frac{-24V\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}-12\cdot 16\phi V}{4\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}^2} = \frac{-24V\left\{\left(2-\frac{3}{2}q\right)^2-1\right\}}{4\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}^2},$$

which is strictly positive since $q < \frac{2}{3}$. So far we have shown two facts. First, when $\phi = \infty$, the maximum value of the left hand side in (20) occurs at $S = \frac{3}{4}V$. In addition, the value of S which achieves the maximum value increases as ϕ decreases. Combining the two, we can conclude that the value of S obtaining the maximum value is alway greater than $\frac{3}{4}V$. Consequently,

$$\frac{-12\phi V - \sqrt{(12\phi V)^2 + 16\phi V^2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}}}{2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}} > \frac{-12\phi V}{2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}} > \frac{3}{4}V.$$

However, $S < \frac{1}{2}V$; otherwise, the price before the arbitrage transaction at s_{23} is $V - 2S < 0$. Therefore, (23) is not valid and we take only the first inequality, (22). As a result, the valid upper bound of S is

$$S^p = \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}}}{2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}}.$$

In conclusion, (i) and (ii) all indicate that the approximate upper bound on S is

$$\overline{S} \simeq S^p = \begin{cases} \frac{-12\phi V + \sqrt{(12\phi V)^2 + 16\phi V^2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}}}{2\left\{\left(2-\frac{3}{2}q\right)^2-1-8\phi\right\}} & \text{if } \left(2-\frac{3}{2}q\right)^2-1-8\phi \lessgtr 0. \\ \frac{V}{3} & \text{if } \left(2-\frac{3}{2}q\right)^2-1-8\phi = 0 \end{cases}$$

Proof of Propositon 4:

Plugging the interior solution to ψ^*_{ti} in Proposition 1 into the following market clearing condition

$$P_{ti} = V - S_{ti} + W_{ti}\left(1 + \psi^*_{ti}\right),$$

and solving for P_{ti} yields the desired result. □

Proof of Proposition 5

Let us rearrange the terms in the square bracket in (15) and evaluate them at the schizophrenic arbitrager's optimal leverage ratio, $\psi^{\text{schizo}}_{ti}$.

$$\underbrace{\frac{E(P_{t+1}|s_{ti})}{P_{ti}} - \left(1 + 2\phi\psi^{\text{schizo}}_{ti}\right)}_{=0} + \frac{\partial\left\{E(P_{t+1}|s_{ti})/P_{ti}\right\}}{\partial\psi_{ti}}(1+\psi_{ti})\Big|_{\psi_{ti}=\psi^{\text{schizo}}_{ti}}$$

Thus, whether the strategic arbitrager assumes more, less, or idential leverage than the schizophrenic arbitrager does depends on the sign of $\frac{\partial \{E(P_{t+1}|s_{ti})/P_{ti}\}}{\partial \psi_{ti}}(1+\psi_{ti})|_{\psi_{ti}=\psi_{ti}^{\text{schizo}}}$.

$$\frac{\partial \left\{E(P_{t+1}|s_{ti})/P_{ti}\right\}}{\partial \psi_{ti}}\Big|_{\psi_{ti}=\psi_{ti}^{\text{schizo}}} = \frac{-E(P_{t+1}|s_{ti})\frac{\partial P_{ti}}{\partial \psi_{ti}}}{P_{ti}^2}\Big|_{\psi_{ti}=\psi_{ti}^{\text{schizo}}}, \tag{24}$$

where

$$\frac{\partial P_{ti}}{\partial \psi_{ti}} = \frac{\partial [V - S_{ti} + W_{ti}(1+\psi_{ti})]}{\partial \psi_{ti}}\Big|_{\psi_{ti}=\psi_{ti}^{\text{schizo}}} = W_{ti} > 0.$$

Therefore $\frac{\partial \{E(P_{t+1}|s_{ti})/P_{ti}\}}{\partial \psi_{ti}}\Big|_{\psi_{ti}=\psi_{ti}^{\text{schizo}}} < 0$, which means

$$\frac{\partial E[W_{t+1}|s_{ti}]}{\partial \psi_{ti}}|_{\psi_{ti}=\psi_{ti}^{\text{schizo}}} < 0.$$

This complete the proof. □

Appendix B: Optimization Problem of the Strategic Arbitrager

Herein we describe the expected terminal wealth of the arbitrager, $E(W_3)$ in 16. At s_{11}, $P_{11} = V$ and $\psi_{11} = 0$. Therefore,

$$\begin{aligned}
W_{3|21|11} &= \underbrace{W_0\left[(1+\psi_0)\frac{V}{P_0} - \psi_0(1+\phi\psi_0)\right]}_{=W_{21|11}=W_{11}} \\
W_{3|22|11} &= \underbrace{W_0\left[(1+\psi_0)\frac{V}{P_0} - \psi_0(1+\phi\psi_0)\right]}_{=W_{22|11}=W_{11}}\left[(1+\psi_{22|11})\frac{V}{P_{22|11}} - \psi_{22|111}(1+\phi\psi_{22|11})\right] \\
W_{3|23|11} &= \underbrace{W_0\left[(1+\psi_0)\frac{V}{P_0} - \psi_0(1+\phi\psi_0)\right]}_{=W_{23|11}=W_{11}}\left[(1+\psi_{23|11})\frac{V}{P_{23|11}} - \psi_{23|111}(1+\phi\psi_{23|11})\right],
\end{aligned}$$

where the relevant prices are from the market clearing conditions,

$$\begin{aligned}
P_{22|11} &= V - S + \underbrace{W_0\left[(1+\psi_0)\frac{V}{P_0} - \psi_0(1+\phi\psi_0)\right]}_{=W_{22|11}=W_{11}}(1+\psi_{22|11}) \\
P_{23|11} &= V - 2S + \underbrace{W_0\left[(1+\psi_0)\frac{V}{P_0} - \psi_0(1+\phi\psi_0)\right]}_{=W_{23|11}=W_{11}}(1+\psi_{23|11}) \\
P_0 &= V - \frac{1}{2}S + W_0(1+\psi_0)
\end{aligned}$$

Similarly we can define the terminal wealths stemming from s_{12}:

$$\begin{aligned}
W_{3|21|12} &= \underbrace{W_0\left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right]}_{W_{21|12}=W_{12}}\left[(1+\psi_{12})\frac{V}{P_{12}} - \psi_{12}(1+\phi\psi_{12})\right] \\
W_{3|22|12} &= \underbrace{W_0\left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right]\left[(1+\psi_{12})\frac{P_{22|12}}{P_{12}} - \psi_{12}(1+\phi\psi_{12})\right]}_{W_{22|12}} \\
&\quad \left[(1+\psi_{22|12})\frac{V}{P_{22|12}} - \psi_{22|12}(1+\phi\psi_{22|12})\right] \\
W_{3|23|12} &= \underbrace{W_0\left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right]\left[(1+\psi_{12})\frac{P_{23|12}}{P_{12}} - \psi_{12}(1+\phi\psi_{12})\right]}_{W_{23|12}} \\
&\quad \left[(1+\psi_{23|12})\frac{V}{P_{23|12}} - \psi_{23|12}(1+\phi\psi_{23|12})\right]
\end{aligned}$$

where

$$\begin{aligned} P_{22|12} &= V - S + W_{22|12}(1+\psi_{22|12}) \\ &= V - S + W_0 \left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right] \left[(1+\psi_{12})\frac{P_{22|12}}{P_{12}} - \psi_{12}(1+\phi\psi_{12})\right](1+\psi_{22|12}) \end{aligned}$$

Solving for $P_{22|12}$ yields

$$P_{22|12} = \frac{V - S - W_0 \left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right] \psi_{12}(1+\phi\psi_{12})(1+\psi_{22|12})}{1 - \frac{W_0 \left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right](1+\psi_{12})(1+\psi_{22|12})}{P_{12}}}$$

Similarly,

$$P_{23|12} = \frac{V - 2S - W_0 \left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right] \psi_{12}(1+\phi\psi_{12})(1+\psi_{23|12})}{1 - \frac{W_0 \left[(1+\psi_0)\frac{P_{12}}{P_0} - \psi_0(1+\phi\psi_0)\right](1+\psi_{12})(1+\psi_{23|12})}{P_{12}}}$$

and

$$P_{12} = \frac{V - S - W_0\psi_0(1+\phi\psi_0)(1+\psi_{12})}{1 - \frac{W_0(1+\psi_0)(1+\psi_{12})}{P_0}}$$

We use the Berndt-Hall-Hall-Hausman algorithm for numerical optimization using a 100 different random combinations of initial values for $\{\psi\}$.

Appendix C: Errors in Variable Problem

We focus on the errors-in-variable of the regressors. EIV leads to a bias in the estimated coefficients, toward zero, which is called 'attenuation bias.' our empirical work, we find that the corrections for this bias are, in general, relatively small; however, they are substantial for the case of non-tail slope spreads. To deal with the EIV problem, we take the following procedures.

(a) Following Theorem 5 of Shanken (1992), we make corrections to the regression coefficients:

$$\hat{\beta}_i = \left[\hat{\mathbb{X}}'\hat{\mathbb{X}} - \begin{pmatrix} 0 & 0 \\ 0 & \sum_{i=1}^{N} \sigma^2(\hat{\delta}_i) \end{pmatrix} \right]^{-1} \hat{\mathbb{X}}' y, \tag{25}$$

where $\mathbb{X} = [1_N, \ x]$ and 1_N is a N-dimensional vector of ones. Note that $x = \{\delta_i\}$, a vector of mean-reversion speeds.

(b) This analytical correction in (a) entails subtracting the sum of squred standard errors of mean-reversion speed from $\hat{\phi}'\hat{\phi}$ to better approximate $\delta'\delta$. However, as shown by Chordia, Goyal and Shanken (2015), this correction may overshoot and the argument inside the squre bracket in (25) may not be positive definite. In case that the matrix in the square bracket fails to be positive definite, we use the instrumental variable approach using higher moments proposed by Dagenais and Dagenais (1997). They introduce an unbiased EIV-corrected estimator, which is a weighted average of the second moment estimator proposed by Durbin (1954) and the third moment estimator proposed by Pal (1970). They show that this new EVI-corrected estimator is more efficient than either of both. Let us denote the Durbin's estimator and the Pal's estimator by $\hat{\beta}_d$ and $\hat{\beta}_h$ respectively. Then,

$$\begin{aligned} \hat{\beta}_{1d} &= (z_1 x)^{-1} z_1' y \\ \hat{\beta}_{1h} &= (z_2 x)^{-1} z_2' y, \end{aligned}$$

where

$$\begin{aligned} x &= \mathcal{A}x \\ y &= \mathcal{A}y \\ \mathcal{A} &= I_N - \frac{\ell_N' \ell_N}{N} \\ z_1 &= x^2 \\ z_2 &= x^3 - 3\frac{x'x}{N}x, \end{aligned}$$

and x^2 and x^3 correspect to the vectors with squares and cubics of each elements of x. ℓ_N is a N-dimensional vector of ones. x and y are demeaned vectors of x and y respectively. Note that both of β_{1d} and β_{1h} are EIV- corrected estimators based on employing the second and the third moments as instrumental variabls respectively. Applying the Generalized Least Square (GLS) method, the resulting EIV-corrected estimator is

$$\hat{\beta}_{1c} = (\ell_2' \Sigma^{-1} \ell_2)^{-1} \ell_2' \Sigma^{-1} \begin{pmatrix} \beta_{1d} \\ \beta_{1h} \end{pmatrix},$$

where Σ is the covariance matrix of β_{1d} and β_{1h} under the null of no measurement errors such that

$$\Sigma = \sigma_{\epsilon}^{2} \begin{bmatrix} (z_1'x)^{-1} z_1' \mathcal{A} z_1 (x'z_1)^{-1} & (z_1'x)^{-1} z_1' \mathcal{A} z_2 (x'z_2)^{-1} \\ (z_1'x)^{-1} z_1' \mathcal{A} z_2 (x'z_2)^{-1} & (z_2'x)^{-1} z_2' \mathcal{A} z_2 (x'z_2)^{-1} \end{bmatrix}.$$

$\hat{\beta}_{1c}$ is unbiased because β_{1d} and β_{1p} are unbiased. The corresponding variance of the estimator is

$$\text{var}(\hat{\beta}_{1c}) = (\ell_2' \Sigma^{-1} \ell_2)^{-1}.$$

Dagenais and Dagenais (1997) show that it is smaller or equal to the smller variance of $\hat{\beta}_{1d}$ and β_{1p}.

The IV mehtod of (b) is designed to correct the bias induced by measurement errors, not specifically tailored to estimation errors. As such, it does not make use of the information on estimation errors of $\hat{x}$, e.g. its standard errors. In that sense, it is less efficient method of correction. In addition, As is true for any kind of IV approach, the instrumental variables, z_1 and z_2 are required to be uncorrelated with error terms but partially and sufficiently strongly correlated with the unobservable x. The first requirement is difficult to be confirmed because we cannot observe the error terms. That is the reason we adopt the above pecking order approach. If (a) is feasible, we employ it and otherwise we adopt (b).

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		Daily									Weekly								
		$p = 0.005$			$p = 0.01$			$p = 0.02$			$p = 0.005$			$p = 0.01$			$p = 0.02$		
Dep. Variable		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		1.647	45.117	0.599	1.647	45.117	0.599	1.647	45.117	0.599	1.551	26.567	0.610	1.551	26.567	0.610	1.551	26.567	0.610
		(51.14)	(10.73)		(51.14)	(10.73)		(51.14)	(10.73)		(43.69)	(10.96)		(43.69)	(10.96)		(43.69)	(10.96)	
VaR	negative	-1.660	-16.319	0.223	-1.639	-8.534	0.127	-1.600	-3.909	0.048	-1.624	-9.233	0.213	-1.613	-5.337	0.126	-1.580	-3.098	0.076
		(-62.45)	(-4.70)		(-84.01)	(-3.35)		(-104.75)	(-1.96)		(-54.87)	(-4.57)		(-68.73)	(-3.33)		(-87.71)	(-2.52)	
	positive	1.805	7.730	0.053	1.728	7.446	0.070	1.644	6.582	0.084	1.780	6.103	0.083	1.690	6.343	0.123	1.611	5.042	0.130
		(63.31)	(2.08)		(72.93)	(2.41)		(86.84)	(2.66)		(51.62)	(2.59)		(59.81)	(3.29)		(73.92)	(3.39)	
Short Fall Risk	negative	-1.675	-32.407	0.473	-1.661	-22.381	0.363	-1.640	-14.223	0.251	-1.619	-17.817	0.404	-1.619	-11.89	0.317	-1.606	-7.994	0.236
		(-56.04)	(-8.31)		(-64.10)	(-6.62)		(-76.48)	(-5.08)		(-44.83)	(-7.23)		(-55.62)	(-5.98)		(-66.78)	(-4.87)	
	positive	1.876	11.498	0.093	1.822	9.444	0.077	1.751	8.269	0.078	1.813	11.317	0.213	1.772	8.999	0.171	1.710	7.321	0.153
		(59.69)	(2.80)		(63.72)	(2.53)		(70.83)	(2.56)		(49.91)	(4.57)		(53.58)	(3.99)		(59.46)	(3.73)	
Tail Volatiilty	negative	-0.019	34.113	0.770	-0.004	27.320	0.845	0.011	21.608	0.892	-0.029	14.309	0.270	-0.018	11.944	0.353	-0.002	9.443	0.408
		(-1.17)	(16.04)		(-0.38)	(20.48)		(1.64)	(25.20)		(-0.73)	(5.34)		(-0.65)	(6.49)		(-0.10)	(7.29)	
	positive	0.051	10.547	0.454	0.068	7.499	0.431	0.087	5.127	0.322	-0.024	11.986	0.455	0.017	8.767	0.449	0.053	6.335	0.419
		(5.06)	(8.01)		(9.00)	(7.64)		(13.40)	(6.05)		(-1.10)	(8.02)		(1.03)	(7.92)		(4.25)	(7.45)	
Non-Tail Volatiilty	negative	0.467	0.251	0.003	0.463	-0.084	0.000	0.454	-0.402	0.012	0.463	0.258	0.009	0.459	0.103	0.002	0.451	-0.130	0.003
		(124.64)	(0.51)		(133.34)	(-0.19)		(140.52)	(-0.95)		(101.13)	(0.83)		(106.52)	(0.35)		(114.07)	(-0.48)	
	positive	0.478	1.023	0.024	0.472	0.833	0.019	0.462	0.547	0.011	0.474	0.791	0.037	0.469	0.660	0.030	0.460	0.415	0.015
		(84.27)	(1.38)		(90.15)	(1.22)		(101.03)	(0.92)		(70.29)	(1.72)		(75.27)	(1.55)		(84.29)	(1.11)	

Table 1A: Relations between Mean-reversion Speed and Other Characteristics of Slopes

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of slope spreads. The EIV bias is not corrected and the mean-reversion speed is estimated by the entire sample.

Dep. Variable		Daily									Weekly								
		$p = 0.005$			**$p = 0.01$**			**$p = 0.02$**			**$p = 0.005$**			**$p = 0.01$**			**$p = 0.02$**		
		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		-2.907	397.144	0.569	-2.907	397.144	0.569	-2.907	397.144	0.569	-6.794	365.054	0.689	-6.794	365.054	0.689	-6.794	365.054	0.689
		(-2.15)	(19.41)		(-2.15)	(19.41)		(-2.15)	(19.41)		(-5.50)	(25.12)		(-5.50)	(25.12)		(-5.50)	(25.12)	
VaR	negative	-2.165	-1.737	0.037	-1.991	-0.386	0.004	-1.809	0.235	0.003	-2.175	-1.188	0.027	-1.997	-0.170	0.001	-1.816	0.356	0.011
		(-62.57)	(-3.32)		(-82.48)	(-1.06)		(-114.13)	(0.98)		(-60.44)	(-2.80)		(-76.65)	(-0.56)		(-105.67)	(1.76)	
	positive	1.883	1.907	0.099	1.826	0.496	0.008	1.756	-0.312	0.004	1.875	0.926	0.037	1.822	0.081	0.000	1.759	-0.471	0.012
		(83.53)	(5.60)		(86.21)	(1.55)		(87.51)	(-1.03)		(78.30)	(3.29)		(81.35)	(0.31)		(83.36)	(-1.90)	
Short Fall Risk	negative	-2.317	-6.073	0.252	-2.196	-3.589	0.141	-2.041	-1.754	0.064	-2.255	-4.353	0.214	-2.165	-2.223	0.092	-2.024	-1.118	0.041
		(-56.54)	(-9.80)		(-63.19)	(-6.83)		(-77.81)	(-4.42)		(-53.66)	(-8.80)		(-61.62)	(-5.38)		(-74.41)	(-3.49)	
	positive	1.951	6.033	0.480	1.905	3.512	0.268	1.848	1.680	0.088	1.918	4.702	0.464	1.883	2.731	0.260	1.841	1.138	0.063
		(79.32)	(16.23)		(83.75)	(10.22)		(87.04)	(5.23)		(75.38)	(15.71)		(81.10)	(10.00)		(83.19)	(4.37)	
Tail Volatiilty	negative	0.156	7.896	0.697	0.172	6.252	0.712	0.201	4.742	0.637	0.049	5.384	0.474	0.105	4.008	0.503	0.149	3.256	0.491
		(7.63)	(25.61)		(11.02)	(26.57)		(14.34)	(22.38)		(1.70)	(16.04)		(5.24)	(17.00)		(8.91)	(16.58)	
	positive	0.065	8.516	0.745	0.072	6.619	0.790	0.082	4.925	0.819	0.009	8.002	0.529	0.030	6.257	0.565	0.056	4.483	0.600
		(3.34)	(28.84)		(5.41)	(32.75)		(9.09)	(35.97)		(0.24)	(17.90)		(1.08)	(19.25)		(3.03)	(20.70)	
Non-Tail Volatiilty	negative	0.509	-0.047	0.002	0.494	-0.089	0.007	0.473	-0.118	0.014	0.512	-0.098	0.010	0.497	-0.136	0.022	0.476	-0.164	0.040
		(107.66)	(-0.66)		(115.06)	(-1.37)		(122.71)	(-2.02)		(101.64)	(-1.66)		(109.09)	(-2.54)		(117.58)	(-3.45)	
	positive	0.502	-0.160	0.011	0.494	-0.215	0.021	0.478	-0.257	0.036	0.501	-0.170	0.018	0.493	-0.210	0.031	0.477	-0.235	0.047
		(83.48)	(-1.76)		(86.16)	(-2.48)		(92.51)	(-3.28)		(80.21)	(-2.31)		(83.12)	(-3.01)		(89.76)	(-3.76)	

Table 1B: Relations between Mean-reversion Speed and Other Characteristics of Butterflies

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of butterfly spreads. The EIV bias is not corrected and the mean-reversion speed is estimated by the entire sample.

		Daily									Weekly								
		$p = 0.005$			$p = 0.01$			$p = 0.02$			$p = 0.005$			$p = 0.01$			$p = 0.02$		
Dep. Variable		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		-2.192	393.430	0.567	-2.192	393.430	0.567	-2.192	393.430	0.567	-5.620	359.667	0.683	-5.620	359.667	0.683	-5.620	359.667	0.683
		(-2.07)	(21.82)		(-2.07)	(21.82)		(-2.07)	(21.82)		(-5.78)	(27.97)		(-5.78)	(27.97)		(-5.78)	(27.97)	
VaR	negative	-2.063	-2.178	0.050	-1.917	-0.696	0.011	-1.765	0.050	0.000	-2.063	-1.604	0.042	-1.918	-0.463	0.007	-1.768	0.177	0.002
		(-70.53)	(-4.38)		(-93.68)	(-2.00)		(-131.76)	(0.22)		(-67.71)	(-3.98)		(-87.02)	(-1.59)		(-122.13)	(0.93)	
	positive	1.871	1.971	0.098	1.810	0.577	0.011	1.737	-0.218	0.001	1.865	0.982	0.038	1.806	0.162	0.001	1.737	-0.374	0.008
		(101.63)	(6.29)		(105.31)	(1.97)		(107.38)	(-0.79)		(95.56)	(3.81)		(99.44)	(0.68)		(102.26)	(-1.67)	
Short Fall Risk	negative	-2.194	-6.624	0.257	-2.090	-4.052	0.152	-1.960	-2.104	0.078	-2.136	-4.816	0.227	-2.058	-2.628	0.110	-1.941	-1.431	0.057
		(-63.12)	(-11.20)		(-70.93)	(-8.08)		(-87.82)	(-5.54)		(-60.48)	(-10.31)		(-69.32)	(-6.69)		(-84.26)	(-4.70)	
	positive	1.939	6.094	0.467	1.891	3.580	0.261	1.832	1.760	0.090	1.909	4.760	0.454	1.872	2.795	0.256	1.824	1.219	0.068
		(96.54)	(17.83)		(101.86)	(11.32)		(106.07)	(5.99)		(92.05)	(17.36)		(98.83)	(11.16)		(101.52)	(5.13)	
Tail Volatiilty	negative	0.140	8.023	0.685	0.151	6.386	0.700	0.173	4.895	0.628	0.052	5.410	0.461	0.095	4.074	0.495	0.128	3.353	0.490
		(8.36)	(28.09)		(11.74)	(29.13)		(14.90)	(24.74)		(2.22)	(17.63)		(5.80)	(18.85)		(9.40)	(18.66)	
	positive	0.064	8.526	0.746	0.072	6.622	0.790	0.084	4.920	0.818	0.011	8.013	0.532	0.033	6.257	0.567	0.060	4.478	0.601
		(4.17)	(32.63)		(6.86)	(36.99)		(11.67)	(40.35)		(0.37)	(20.32)		(1.51)	(21.82)		(4.11)	(23.37)	
Non-Tail Volatiilty	negative	0.499	-0.010	0.000	0.487	-0.061	0.003	0.468	-0.100	0.010	0.501	-0.060	0.003	0.489	-0.106	0.013	0.470	-0.144	0.030
		(129.56)	(-0.15)		(139.89)	(-1.02)		(151.19)	(-1.90)		(121.51)	(-1.10)		(131.69)	(-2.15)		(143.99)	(-3.34)	
	positive	0.498	-0.140	0.008	0.490	-0.196	0.017	0.475	-0.243	0.032	0.497	-0.152	0.014	0.490	-0.194	0.026	0.475	-0.224	0.042
		(103.13)	(-1.71)		(106.70)	(-2.51)		(115.09)	(-3.45)		(99.30)	(-2.30)		(103.12)	(-3.09)		(111.86)	(-3.99)	

Table 1C: Relations between Mean-reversion Speed and Other Characteristics of All Spreads

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of all spreads: slopes and butterflies. The EIV bias is not corrected and the mean-reversion speed is estimated by the entire sample.

		Daily									Weekly								
		$p = 0.005$			**$p = 0.01$**			**$p = 0.02$**			**$p = 0.005$**			**$p = 0.01$**			**$p = 0.02$**		
Dep. Variable		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		1.639	47.119	0.598	1.639	47.119	0.598	1.639	47.119	0.598	1.480	33.222	0.571	1.480	33.222	0.571	1.480	33.222	0.571
		(50.80)	(11.19)		(50.80)	(11.19)		(50.80)	(11.19)		(39.79)	(13.08)		(39.79)	(13.08)		(39.79)	(13.08)	
VaR	negative	-1.657	-17.043	0.223	-1.637	-8.912	0.127	-1.599	-4.083	0.047	-1.599	-11.546	0.200	-1.599	-6.675	0.118	-1.572	-3.874	0.071
		(-55.78)	(-8.67)		(-63.90)	(-6.91)		(-76.33)	(-5.31)		(-42.62)	(-8.85)		(-53.76)	(-7.37)		(-65.27)	(-6.03)	
	positive	1.804	8.073	0.053	1.727	7.776	0.070	1.642	6.874	0.084	1.764	7.632	0.075	1.673	7.932	0.115	1.598	6.305	0.122
		(63.26)	(2.17)		(72.87)	(2.51)		(86.77)	(2.78)		(51.01)	(3.23)		(58.95)	(4.10)		(72.96)	(4.22)	
Short Fall Risk	negative	-1.669	-33.845	0.472	-1.657	-23.374	0.362	-1.637	-14.855	0.251	-1.572	-22.280	0.379	-1.588	-14.866	0.297	-1.585	-9.996	0.221
		(-55.78)	(-8.67)		(-63.90)	(-6.91)		(-76.33)	(-5.31)		(-42.62)	(-8.85)		(-53.76)	(-7.37)		(-65.27)	(-6.03)	
	positive	1.874	12.008	0.092	1.821	9.863	0.077	1.749	8.636	0.078	1.782	14.152	0.200	1.748	11.254	0.160	1.690	9.155	0.143
		(59.61)	(2.93)		(63.65)	(2.64)		(70.76)	(2.68)		(48.67)	(5.66)		(52.51)	(4.95)		(58.45)	(4.64)	
Tail Volatiilty	negative	-0.026	35.627	0.768	-0.009	28.533	0.843	0.007	22.567	0.890	-0.067	17.894	0.253	-0.049	14.936	0.331	-0.027	11.808	0.383
		(-1.57)	(16.70)		(-0.89)	(21.28)		(1.00)	(26.11)		(-1.68)	(6.60)		(-1.80)	(7.97)		(-1.39)	(8.92)	
	positive	0.049	11.015	0.454	0.066	7.831	0.430	0.086	5.355	0.322	-0.056	14.989	0.427	-0.007	10.963	0.421	0.036	7.923	0.393
		(4.85)	(8.36)		(8.81)	(7.97)		(13.25)	(6.32)		(-2.49)	(9.78)		(-0.40)	(9.66)		(2.83)	(9.12)	
Non-Tail Volatiilty	negative	0.467	0.262	0.003	0.463	-0.088	0.000	0.454	-0.419	0.012	0.462	0.323	0.011	0.459	0.129	0.035	0.452	-0.163	0.015
		(124.63)	(0.54)		(133.35)	(-0.19)		(140.55)	(-0.99)		(100.95)	(1.03)		(106.45)	(0.44)		(114.15)	(-0.60)	
	positive	0.478	1.068	0.024	0.472	0.870	0.019	0.462	0.571	0.011	0.472	0.989	0.035	0.467	0.825	0.028	0.459	0.519	0.015
		(84.23)	(1.44)		(90.12)	(1.27)		(101.01)	(0.96)		(69.89)	(2.14)		(74.91)	(1.94)		(84.04)	(1.39)	

Table 2A: Relations between Mean-reversion Speed and Other Characteristics of Slopes: EIV-Adjusted

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of slope spreads. The EIV bias is corrected and the mean-reversion speed is estimated by the entire sample.

		Daily									Weekly								
		$p = 0.005$			**$p = 0.01$**			**$p = 0.02$**			**$p = 0.005$**			**$p = 0.01$**			**$p = 0.02$**		
Dep. Variable		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		-2.924	397.930	0.569	-2.924	397.930	0.569	2.924	397.930	0.569	-6.923	368.820	0.689	-6.923	368.820	0.689	-6.923	368.820	0.689
		(-2.16)	(19.45)		(-2.16)	(19.45)		(-2.16)	(19.45)		(-5.60)	(25.37)		(-5.60)	(25.37)		(-5.60)	(25.37)	
VaR	negative	-2.165	-1.741	0.037	-1.991	-0.387	0.004	-1.809	0.235	0.003	-2.175	-1.200	0.027	-1.997	-0.172	0.001	-1.816	0.360	0.011
		(-62.57)	(-3.33)		(-82.48)	(-1.06)		(-114.13)	(0.98)		(-60.43)	(-2.83)		(-76.64)	(-0.56)		(-105.68)	(1.78)	
	positive	1.883	1.911	0.099	1.826	0.497	0.008	1.756	-0.312	0.004	1.875	0.936	0.037	1.822	0.082	0.000	1.759	-0.476	0.012
		(83.53)	(5.61)		(86.21)	(1.55)		(87.51)	(-1.03)		(78.28)	(3.32)		(81.35)	(0.31)		(83.37)	(-1.92)	
Short Fall Risk	negative	-2.317	-6.085	0.252	-2.195	-3.596	0.141	-2.041	-1.758	0.064	-2.254	-4.398	0.214	-2.164	-2.246	0.092	-2.024	-1.130	0.041
		-(56.53)	(-9.82)		(-63.19)	(-6.85)		(-77.81)	(-4.43)		(-53.62)	(-8.89)		(-61.59)	(-5.43)		(-74.39)	(-3.53)	
	positive	1.950	6.045	0.480	1.905	3.519	0.268	1.848	1.683	0.088	1.916	4.750	0.464	1.882	2.759	0.260	1.840	1.150	0.063
		(79.31)	(16.26)		(83.74)	(10.24)		(87.04)	(5.24)		(75.31)	(15.87)		(81.06)	(10.10)		(83.17)	(4.42)	
Tail Volatiilty	negative	0.155	7.912	0.697	0.171	6.265	0.712	0.201	4.752	0.637	0.047	5.440	0.474	0.104	4.049	0.503	0.148	3.290	0.491
		(7.61)	(25.66)		(11.01)	(26.62)		(14.32)	(22.42)		(1.63)	(16.20)		(5.17)	(17.17)		(8.84)	(16.75)	
	positive	0.065	8.532	0.745	0.072	6.632	0.790	0.082	4.935	0.819	0.006	8.085	0.529	0.028	6.322	0.565	0.054	4.530	0.600
		(3.32)	(28.89)		(5.39)	(32.81)		(9.07)	(36.04)		(0.17)	(18.08)		(1.00)	(19.45)		(2.94)	(20.91)	
Non-Tail Volatiilty	negative	0.509	-0.047	0.002	0.494	-0.089	0.007	0.473	-0.118	0.014	0.512	-0.099	0.010	0.497	-0.137	0.022	0.476	-0.166	0.040
		(107.66)	(-0.66)		(115.06)	(-1.37)		(122.71)	(-2.02)		(101.64)	(-1.68)		(109.10)	(-2.56)		(117.60)	(-3.49)	
	positive	0.502	-0.161	0.011	0.494	-0.215	0.021	0.478	-0.257	0.036	0.501	-0.172	0.018	0.493	-0.212	0.031	0.478	-0.238	0.047
		(83.49)	(-1.77)		(86.16)	(-2.48)		(92.51)	(-3.29)		(80.22)	(-2.33)		(83.13)	(-3.04)		(89.78)	(-3.80)	

Table 2B: Relations between Mean-reversion Speed and Other Characteristics of Butterflies: EIV-Adjusted
This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of butterfly spreads. The EIV bias is corrected and the mean-reversion speed is estimated by the entire sample.

		Daily									Weekly								
		$p = 0.005$			**$p = 0.01$**			**$p = 0.02$**			**$p = 0.005$**			**$p = 0.01$**			**$p = 0.02$**		
Dep. Variable		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		-2.206	394.240	0.567	-2.206	394.240	0.567	-2.206	394.240	0.567	-5.736	363.616	0.683	-5.736	363.616	0.683	-5.736	363.616	0.683
		(-2.08)	(21.87)		(-2.08)	(21.87)		(-2.08)	(21.87)		(-5.90)	(28.27)		(-5.90)	(28.27)		(-5.90)	(28.27)	
VaR	negative	-2.063	-2.183	0.050	-1.917	-0.697	0.011	-1.765	0.050	0.000	-2.063	-1.622	0.042	-1.918	-0.468	0.007	-1.768	0.179	0.002
		(-70.53)	(-4.38)		(-93.68)	(-2.00)		(-131.76)	(0.22)		(-67.69)	(-4.03)		(-87.02)	(-1.61)		(-122.13)	(0.94)	
	positive	1.871	1.975	0.098	1.810	0.578	0.011	1.737	-0.218	0.002	1.865	0.993	0.038	1.806	0.164	0.001	1.737	-0.378	0.008
		(101.62)	(6.30)		(105.31)	(1.98)		(107.38)	(-0.79)		(95.54)	(3.85)		(99.43)	(0.68)		(102.27)	(-1.68)	
Short Fall Risk	negative	-2.194	-6.638	0.257	-2.090	-4.060	0.152	-1.960	-2.108	0.078	-2.135	-4.869	0.227	-2.057	-2.657	0.110	-1.941	-1.447	0.057
		(-63.12)	(-11.22)		(-70.93)	(-8.09)		(-87.81)	(-5.55)		(-60.44)	(-10.43)		(-69.29)	(-6.77)		(-84.24)	(-4.75)	
	positive	1.938	6.107	0.467	1.891	3.587	0.261	1.832	1.764	0.090	1.907	4.812	0.454	1.871	2.826	0.256	1.824	1.232	0.068
		(96.53)	(17.86)		(101.85)	(11.35)		(106.06)	(6.00)		(91.98)	(17.55)		(98.78)	(11.29)		(101.49)	(5.19)	
Tail Volatiilty	negative	0.140	8.040	0.685	0.151	6.400	0.700	0.173	4.905	0.628	0.050	5.469	0.461	0.094	4.119	0.495	0.127	3.390	0.489
		(8.34)	(28.15)		(11.72)	(29.19)		(14.88)	(24.79)		(2.15)	(17.82)		(5.72)	(19.06)		(9.32)	(18.66)	
	positive	0.064	8.544	0.746	0.072	6.636	0.790	0.083	4.930	0.818	0.008	8.101	0.532	0.031	6.326	0.567	0.058	4.527	0.601
		(4.15)	(32.70)		(6.83)	(37.06)		(11.65)	(40.43)		(0.28)	(20.54)		(1.42)	(22.06)		(4.01)	(23.62)	
Non-Tail Volatiilty	negative	0.499	-0.010	0.000	0.487	-0.061	0.003	0.468	-0.100	0.010	0.501	-0.061	0.003	0.489	-0.107	0.013	0.470	-0.146	0.030
		(129.56)	(-0.15)		(139.89)	(-1.02)		(151.19)	(-1.90)		(121.51)	(-1.11)		(131.70)	(-2.18)		(144.01)	(-3.38)	
	positive	0.498	-0.140	0.008	0.490	-0.197	0.017	0.475	-0.243	0.032	0.497	-0.154	0.014	0.490	-0.196	0.026	0.475	-0.226	0.042
		(103.13)	(-1.71)		(106.70)	(-2.51)		(115.09)	(-3.46)		(99.31)	(-2.32)		(103.13)	(-3.12)		(111.88)	(-4.03)	

Table 2C: Relations between Mean-reversion Speed and Other Characteristics of All Spreads: EIV-Adjusted

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of all spreads: slopes and butterflies. The EIV bias is corrected and the mean-reversion speed is estimated by the entire sample.

		Daily									Weekly								
		$p = 0.005$			**$p = 0.01$**			**$p = 0.02$**			**$p = 0.005$**			**$p = 0.01$**			**$p = 0.02$**		
Dep. Variable		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		1.666	79.807	0.353	1.669	92.831	0.311	1.716	87.088	0.209	1.484	50.734	0.425	1.551	45.411	0.345	1.551	52.553	0.405
		(38.44)	(6.49)		(36.52)	(5.89)		(36.73)	(4.52)		(27.01)	(7.54)		(28.52)	(6.37)		(31.48)	(7.24)	
VaR	negative	-1.619	-50.490	0.402	-1.629	-25.056	0.134	-1.606	-7.720	0.017	-1.527	-28.429	0.387	-1.592	-12.427	0.132	-1.572	-7.531	0.076
		(-65.57)	(-7.20)		(-77.54)	(-3.46)		(-100.26)	(-1.17)		(-45.80)	(-6.97)		(-57.55)	(-3.43)		(-77.65)	(-2.52)	
	positive	1.771	30.667	0.157	1.691	37.334	0.215	1.616	38.968	0.277	1.684	23.467	0.227	1.634	19.792	0.233	1.565	18.630	0.301
		(62.17)	(3.79)		(71.83)	(4.60)		(93.08)	(5.44)		(41.75)	(4.75)		(52.25)	(4.83)		(71.23)	(5.75)	
Short Fall Risk	negative	-1.649	-75.605	0.484	-1.651	-57.052	0.288	-1.657	-30.448	0.108	-1.539	-39.186	0.374	-1.609	-22.035	0.211	-1.607	-15.607	0.152
		(-52.67)	(-8.51)		(-55.77)	(-5.58)		(93.08)	(5.44)		(-32.58)	(-6.78)		(-43.45)	(-4.54)		(-56.45)	(-3.72)	
	positive	1.841	38.409	0.195	1.781	43.761	0.202	1.717	47.948	0.248	1.704	33.263	0.352	1.709	25.625	0.269	1.650	25.632	0.318
		(58.70)	(4.31)		(61.94)	(4.41)		(74.46)	(5.04)		(40.53)	(6.46)		(46.48)	(5.32)		(56.88)	(6.00)	
Tail Volatiilty	negative	0.033	43.171	0.232	0.027	46.382	0.298	0.053	35.030	0.220	0.050	10.752	0.029	0.043	10.614	0.054	0.052	8.668	0.058
		(1.05)	(4.83)		(1.17)	(5.72)		(2.93)	(4.66)		(0.86)	(1.52)		(1.13)	(2.10)		(1.93)	(2.18)	
	positive	0.065	14.139	0.154	0.073	14.525	0.198	0.092	11.987	0.165	-0.043	21.252	0.274	0.013	15.627	0.276	0.044	14.245	0.360
		(4.92)	(3.74)		(7.56)	(4.36)		(12.34)	(3.90)		(-1.32)	(5.38)		(0.59)	(5.42)		(2.98)	(6.57)	
Non-Tail Volatiilty	negative	0.464	1.849	0.035	0.462	0.227	0.000	0.454	-1.237	0.010	0.459	0.984	0.024	0.459	0.247	0.002	0.451	-0.267	0.002
		(118.82)	(1.67)		(123.11)	(0.18)		(136.06)	(-0.90)		(79.23)	(1.39)		(89.94)	(0.37)		(101.43)	(-0.41)	
	positive	0.472	4.952	0.107	0.465	5.828	0.113	0.456	5.955	0.120	0.456	3.902	0.171	0.456	3.271	0.144	0.447	3.139	0.154
		(82.02)	(3.03)		(86.34)	(3.14)		(102.35)	(3.24)		(57.11)	(3.99)		(65.79)	(3.61)		(78.68)	(3.75)	

Table 3A: Relations between Mean-reversion Speed and Other Characteristics of Slopes: Revisted

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of slope spreads. The EIV bias is not corrected and the mean-reversion speed is estimated by tranquil time periods only.

		Daily									Weekly								
		$p = 0.005$			**$p = 0.01$**			**$p = 0.02$**			**$p = 0.005$**			**$p = 0.01$**			**$p = 0.02$**		
Dep. Variable		β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2	β_{0i}	β_{1i}	R^2
Kurtosis		-3.842	907.468	0.525	-4.389	1077.311	0.526	-4.723	1295.127	0.480	-7.944	698.365	0.496	-8.343	790.972	0.496	-10.115	967.720	0.496
		(-2.66)	(17.74)		(-3.02)	(17.80)		(-3.06)	(16.23)		(-4.80)	(16.74)		(-5.01)	(16.74)		(-5.87)	(16.74)	
VaR	negative	-2.143	-5.689	0.071	-1.969	-3.205	0.034	-1.795	-1.172	0.007	-2.138	-3.987	0.060	-1.968	-1.987	0.023	-1.787	-1.039	0.009
		(-61.96)	(-4.65)		(-80.81)	(-3.16)		(-109.27)	(-1.38)		(-57.44)	(-4.25)		(-72.05)	(-2.57)		(-94.88)	(-1.64)	
	positive	1.870	5.211	0.130	1.812	2.693	0.031	1.739	1.253	0.005	1.842	3.311	0.092	1.795	1.684	0.022	1.726	1.033	0.006
		(82.99)	(6.54)		(84.40)	(3.02)		(83.56)	(1.16)		(75.30)	(5.37)		(76.46)	(2.53)		(74.46)	(1.33)	
Short Fall Risk	negative	-2.286	-15.453	0.288	-2.156	-12.545	0.216	-2.001	-9.808	0.159	-2.169	-12.058	0.322	-2.102	-7.805	0.174	-1.953	-6.673	0.150
		(-56.21)	(-10.74)		(-63.38)	(-8.87)		(-77.53)	(-7.35)		(-52.83)	(-11.65)		(-59.21)	(-7.75)		(-69.64	(-7.09)	
	positive	1.934	14.058	0.460	1.884	10.358	0.293	1.820	8.005	0.158	1.898	9.248	0.353	1.857	6.757	0.244	1.790	5.463	0.148
		(75.85)	(15.59)		(82.21)	(10.87)		(85.98)	(7.31)		(64.54)	(12.47)		(74.66)	(9.58)		(77.51)	(7.05)	
Tail Volatiilty	negative	0.146	17.128	0.579	0.152	16.524	0.625	0.170	16.657	0.624	-0.035	13.723	0.606	0.043	11.205	0.603	0.096	10.033	0.477
		(5.99)	(19.80)		(8.36)	(21.81)		(11.46)	(21.74)		(-1.36)	(20.94)		(2.27)	(20.83)		(5.20)	(16.13)	
	positive	0.057	18.355	0.611	0.059	16.699	0.632	0.066	15.285	0.626	0.057	11.575	0.218	0.057	10.540	0.246	0.039	10.387	0.330
		(2.31)	(21.15)		(3.26)	(22.13)		(4.88)	(21.86)		(1.11)	(8.91)		(1.48)	(9.64)		(1.51)	(11.85)	
Non-Tail Volatiilty	negative	0.507	0.058	0.000	0.493	-0.041	0.000	0.472	-0.191	0.003	0.509	-0.022	0.000	0.495	-0.130	0.003	0.474	-0.221	0.007
		(105.43)	(0.34)		(111.52)	(-0.22)		(117.31)	(-0.92)		(95.60)	(-0.17)		(101.50)	(-0.94)		(105.15)	(-1.46)	
	positive	0.501	-0.208	0.003	0.493	-0.328	0.006	0.477	-0.473	0.010	0.498	-0.148	0.003	0.490	-0.229	0.006	0.474	-0.303	0.008
		(81.55)	(-0.96)		(83.15)	(-1.33)		(87.60)	(-1.68)		(75.20)	(-0.88)		(76.96)	(-1.27)		(79.81)	(-1.52)	

Table 3B: Relations between Mean-reversion Speed and Other Characteristics of Butterflies: Revisted

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of butterflies. The EIV bias is not corrected and the mean-reversion speed is estimated by tranquil time periods only.

Dep. Variable		Daily $p=0.005$ β_{0i}	β_{1i}	R^2	Daily $p=0.01$ β_{0i}	β_{1i}	R^2	Daily $p=0.02$ β_{0i}	β_{1i}	R^2
Kurtosis		-2.955	896.021	0.522	-3.366	1061.608	0.523	-3.563	1271.088	0.476
		(-2.61)	(19.92)		(-2.96)	(19.96)		(-2.96)	(18.17)	
		(-2.07)	(21.82)		(-2.07)	(21.82)		(-2.07)	(21.82)	
VaR	negative	-2.042	-6.864	0.088	-1.897	-4.194	0.050	-1.751	-1.968	0.016
		(-70.21)	(-5.93)		(-92.61)	(-4.37)		(-127.79)	(-2.47)	
	positive	1.859	5.388	0.130	1.798	2.961	0.035	1.722	1.671	0.008
		(101.31)	(7.38)		(103.69)	(3.65)		(103.55)	(1.73)	
Short Fall Risk	negative	-2.166	-16.868	0.295	-2.055	-13.974	0.229	-1.924	-11.227	0.178
		(-63.04)	(-12.33)		(-71.60)	(-10.40)		(-88.52)	(-8.87)	
	positive	1.924	14.223	0.451	1.874	10.570	0.288	1.808	8.327	0.162
		(93.01)	(17.27)		(100.65)	(12.12)		(105.79)	(8.37)	
Tail Volatiilty	negative	0.133	17.341	0.568	0.135	16.817	0.615	0.148	17.102	0.616
		(6.65)	(21.83)		(9.07)	(24.08)		(12.13)	(24.14)	
	positive	0.057	18.344	0.612	0.062	16.663	0.634	0.071	15.184	0.626
		(2.94)	(23.95)		(4.33)	(25.06)		(6.72)	(24.66)	
Non-Tail Volatiilty	negative	0.498	0.159	0.003	0.486	0.054	0.000	0.467	-0.110	0.001
		(127.41)	(1.02)		(136.35)	(0.32)		(145.61)	(-0.59)	
	positive	0.497	-0.150	0.002	0.489	-0.262	0.004	0.474	-0.403	0.007
		(101.03)	(-0.77)		(103.45)	(-1.19)		(109.80)	(-1.60)	

Dep. Variable		Weekly $p=0.005$ β_{0i}	β_{1i}	R^2	Weekly $p=0.01$ β_{0i}	β_{1i}	R^2	Weekly $p=0.02$ β_{0i}	β_{1i}	R^2
Kurtosis		-6.646	684.588	0.491	-6.944	773.254	0.490	-8.233	936.653	0.486
		(-5.14)	(18.71)		(-5.33)	(18.67)		(-6.13)	(18.53)	
		(-5.78)	(27.97)		(-5.78)	(27.97)		(-5.78)	(27.97)	
VaR	negative	-2.025	-5.072	0.083	-1.888	-2.846	0.040	-1.739	-1.738	0.024
		(-64.73)	(-5.73)		(-82.55)	(-3.91)		(-111.87)	(-2.97)	
	positive	1.836	3.441	0.093	1.781	1.909	0.027	1.706	1.399	0.011
		(92.33)	(6.12)		(94.14)	(3.17)		(92.75)	(2.02)	
Short Fall Risk	negative	-2.055	-13.154	0.335	-1.998	-8.927	0.197	-1.873	-7.818	0.180
		(-59.82)	(-13.53)		(-67.11)	(-9.43)		(-80.33)	(-8.91)	
	positive	1.889	9.414	0.352	1.847	6.941	0.245	1.780	5.724	0.156
		(79.73)	(14.05)		(91.71)	(10.84)		(95.95)	(8.21)	
Tail Volatiilty	negative	-0.019	13.568	0.575	0.042	11.211	0.581	0.083	10.210	0.476
		(-0.88)	(22.18)		(2.69)	(22.44)		(5.54)	(18.17)	
	positive	0.048	11.690	0.225	0.053	10.600	0.253	0.046	10.324	0.335
		(1.19)	(10.26)		(1.77)	(11.08)		(2.25)	(13.52)	
Non-Tail Volatiilty	negative	0.498	0.077	0.001	0.486	-0.040	0.000	0.468	-0.138	0.003
		(114.97)	(0.63)		(123.35)	(-0.32)		(130.25)	(-1.02)	
	positive	0.494	-0.098	0.001	0.487	-0.176	0.003	0.472	-0.248	0.005
		(93.49)	(-0.65)		(95.99)	(-1.09)		(100.51)	(-1.40)	

Table 3C: Relations between Mean-reversion Speed and Other Characteristics of All Spreads: Revisted

This table reports the estimation results of cross-sectional regressions between mean-reversion speed and tail risk measures/tail and non-tail volatility measures of all spreads: slopes and butterflies. The EIV bias is not corrected and the mean-reversion speed is estimated by tranquil time periods only.

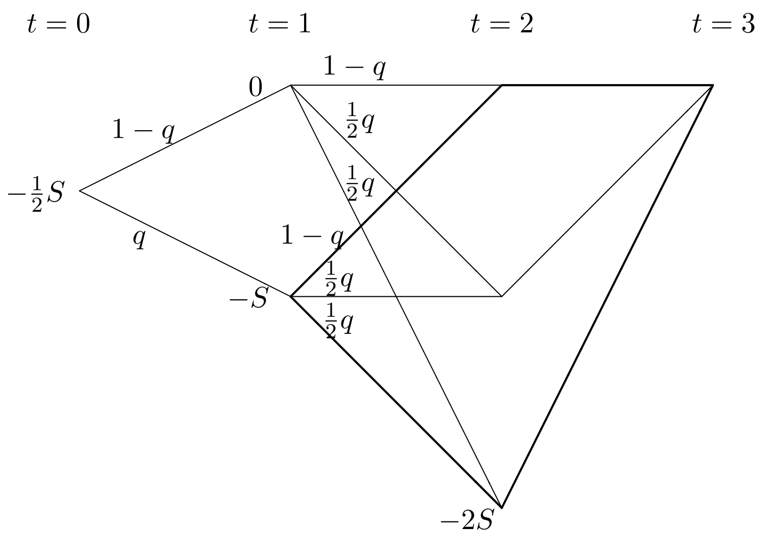


Figure 1: State Space of a pessimistic shock of noise traders S.
This figure illustrates the state space of S, a time-series evolution of states on S, which is an amount of deviation from the fundamental value, V, due to a pessimistic misconception of noise traders. The thick line corresponds to the state space of S used in the model of Shleifer and Vishny (1997), which is a special case of our model.

$t=0$	$t=1$	$t=2$	$t=3$
	$\psi_{11}=-1$ $P_{11}=1$ $W_{11}=0.0538$	$\psi_{21\|11}=\psi_{21\|12}=-1$ $P_{21\|11}=P_{21\|12}=1$ $W_{21\|11}=0.0538$ $W_{21\|12}=0.0552$	$W_{3\|21\|11}=0.0538$ $W_{3\|21\|12}=0.0552$
$\psi_0=0.2799$ $P_0=0.9390$ $W_0=0.0500$			
	$\psi_{12}=0.9229$ $P_{12}=0.8313$ $W_{12}=0.0423$	$\psi_{22\|11}=0.8763$ $P_{22\|11}=0.8509$ $W_{22\|11}=0.0538$	$W_{3\|22\|11}=0.0673$
		$\psi_{22\|12}=1.0375$ $P_{22\|12}=0.8282$ $W_{22\|12}=0.0384$	$W_{3\|22\|12}=0.0505$
		$\psi_{23\|11}=2.3511$ $P_{23\|11}=0.6802$ $W_{23\|11}=0.0538$	$W_{3\|23\|11}=0.1088$
		$\psi_{23\|12}=3.9318$ $P_{23\|12}=0.5598$ $W_{23\|12}=0.0121$	$W_{3\|23\|12}=0.0404$

Figure 2(a): Optimal Leverage Ratios, Equilibrium Prices, Arbitrager's Wealth: Schizophrenic Case

This figure illustrates the optimal leverage ratio, ψ, the equilibrium price, P, and the corresponding wealth of the arbitrager, W, across states and time in the schizophrenic arbitrager model. Structural paramers used are $V=1$, $W_0=0.05$, $S=0.25$, $\phi=0.1$ and $q=0.05$.

$t=0$	$t=1$	$t=2$	$t=3$
	$\psi_{11}=-1$ $P_{11}=1$ $W_{11}=0.0541$	$\psi_{21\|11}=\psi_{21\|12}=-1$ $P_{21\|11}=P_{21\|12}=1$ $W_{21\|11}=0.0541$ $W_{21\|12}=0.0580$	$W_{3\|21\|11}=0.0541$ $W_{3\|21\|12}=0.0580$
$\psi_0=0.0380$ $P_0=0.9269$ $W_0=0.0500$			
	$\psi_{12}=0.6753$ $P_{12}=0.8241$ $W_{12}=0.0442$	$\psi_{22\|11}=0.4583$ $P_{22\|11}=0.8289$ $W_{22\|11}=0.0541$ $\psi_{22\|12}=0.9613$ $P_{22\|12}=0.8347$ $W_{22\|12}=0.0431$	$W_{3\|22\|11}=0.0692$ $W_{3\|22\|12}=0.0560$
		$\psi_{23\|11}=1.3534$ $P_{23\|11}=0.6273$ $W_{23\|11}=0.0541$ $\psi_{23\|12}=3.3542$ $P_{23\|12}=0.5936$ $W_{23\|12}=0.0215$	$W_{3\|23\|11}=0.1198$ $W_{3\|23\|12}=0.0614$

Figure 2(b): Optimal Leverage Ratios, Equilibrium Prices, Arbitrager's Wealth: Strategic Case

This figure illustrates the optimal leverage ratio, ψ, the equilibrium price, P, and the corresponding wealth of the arbitrager, W, across states and time in the strategic arbitrager model. Structural paramers used are $V = 1$, $W_0 = 0.05$, $S = 0.25$, $\phi = 0.1$ and $q = 0.05$.

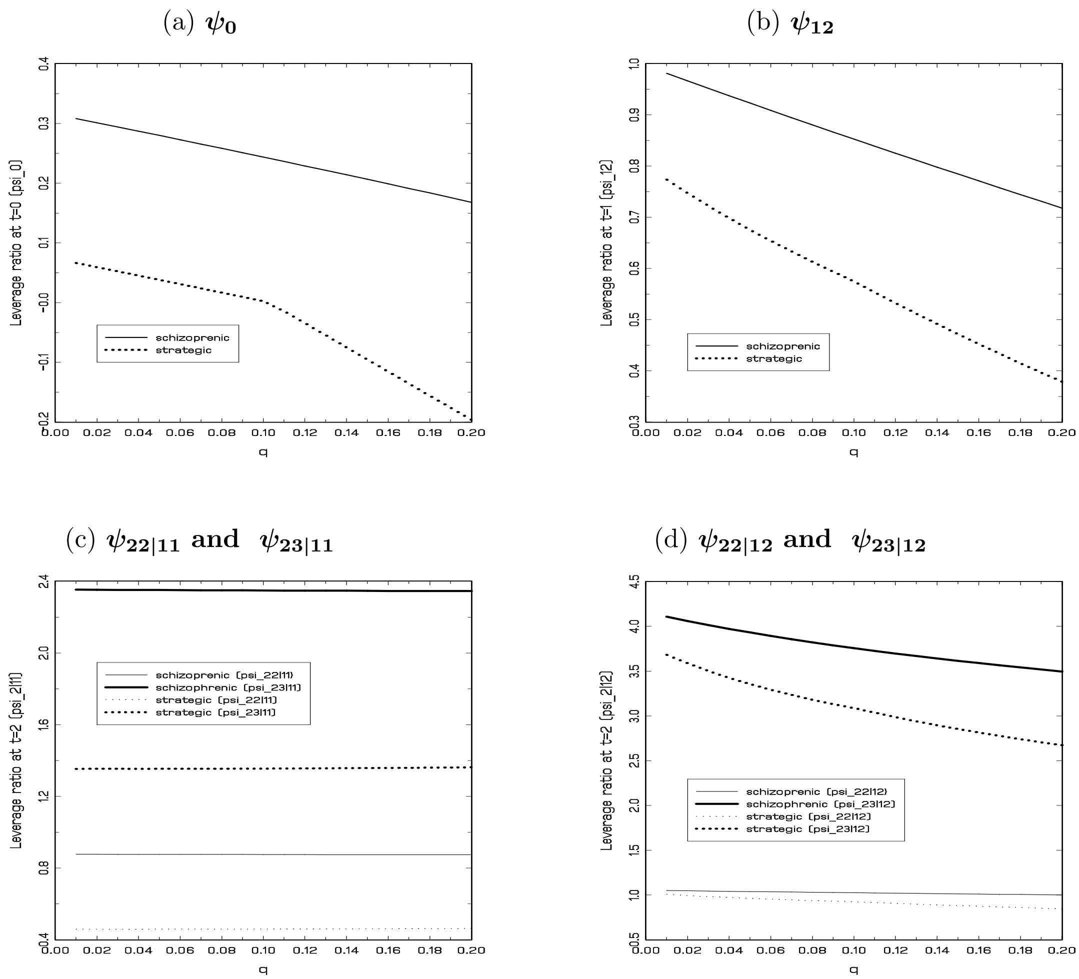


Figure 3: Optimal Leverage Ratios: ψ.
This figure illustrates the optimal leverage ratios, ψs (from $t = 0$ to $t = 2$) as a function of the probability of pessimistic negative shock, q. $\psi_{11} = \psi_{21|11} = \psi_{21|12} = -1$ so that they were not drawn.

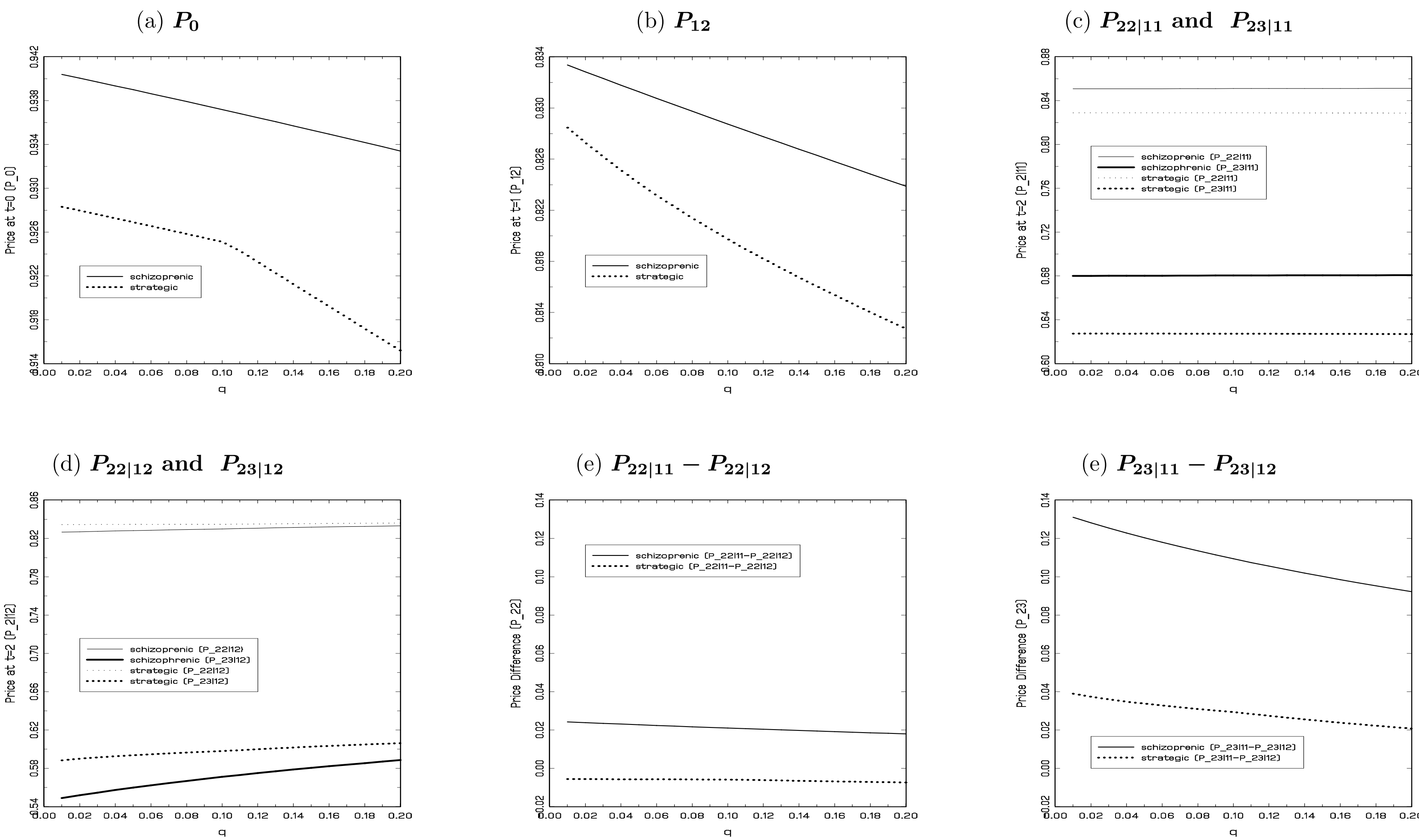
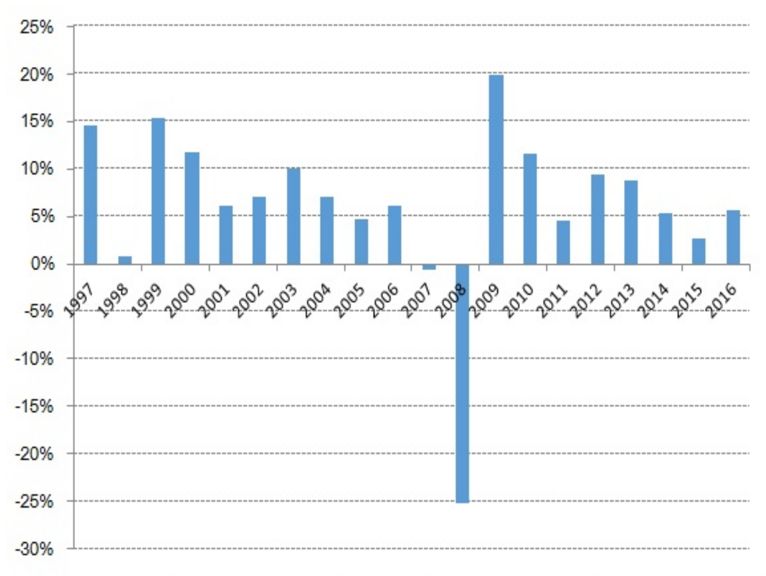


Figure 4: Equilibrium Prices: P.

This figure illustrates the equilibrium prices, Ps (from $t = 0$ to $t = 2$) as a function of the probability of pessimistic negative shock, q. (c) and (d) depict the price differentials driven by previous path at $s_1 1$ and $s_1 2$ respectively. $P_{11} = P_{21|11} = P_{21|12} = V$ so that they were not drawn.

(a) Annual Returns of Fixed Income Arbitrage Funds



(b) Distribution of Monthly Returns of Fixed Income Arbitrage Funds

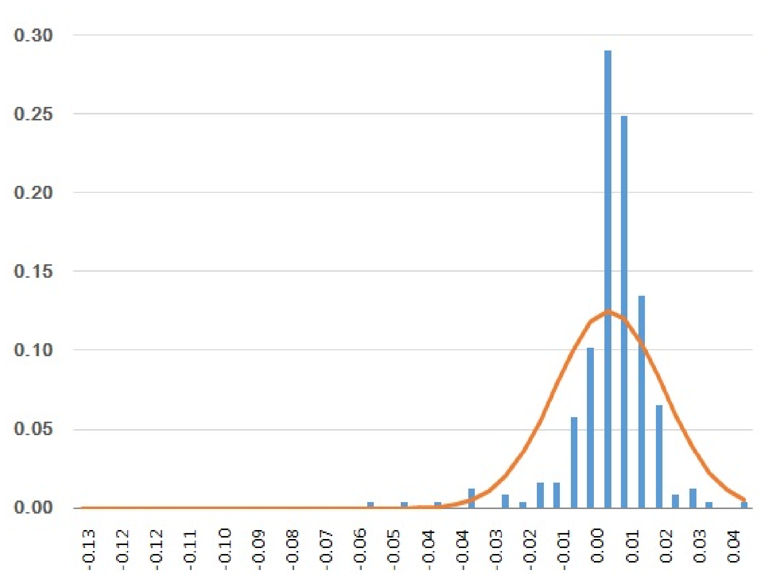


Figure 5: Returns of Fixed Income Arbitrage Funds.
(a) illustrates historical annual returns of fixed income arbitrage funds. (b) shows the probability distribution of the monthly returns of fixed income arbitrage funds coupled with its corresponding normal distribution.