Does Corporate Investment Respond to Time-Varying Risk? Empirical Evidence

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Abstract

I test whether firms take time-varying risk into account in their capital budgeting decisions. The challenge to this test lies in measuring a conditional cost of equity. To do so, I nonparametrically estimate the firm-level equity premium implicit in individual equity option prices. The empirical analysis establishes that corporate investment responds negatively to fluctuations in the equity premium. This finding suggests that managers adjust their cost of capital correctly to time-varying risk, beyond depending entirely on the Capital Asset Pricing Model. I also provide empirical evidence supporting the hypothesis that time-varying risk causes corporate investment to negatively predict subsequent stock returns.

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1 Introduction

Determining the cost of capital is essential to a firm's capital budgeting decisions. Despite its importance, however, a discrepancy exists in characterizing the cost of capital, particularly the cost of equity, between modern asset pricing studies and corporate practices. A growing number of studies on asset pricing reveal time-variation in the equity premium (e.g. Campbell and Cochrane (1999); Bansal and Yaron (2004); Cochrane (2011)). However, most corporate finance textbooks have not kept up with this understanding of time-varying risk. The survey by Graham and Harvey (2001) also reports that 74% of CFOs determine the cost of equity using the Capital Asset Pricing Model. Nevertheless, the CAPM and other existing factor models as well only poorly capture time-variation in cost of capital as found by Fama and French (1997), Welch and Goyal (2008), and Levi and Welch (2016).[*]

Observing this inconsistency, I ask whether firms really ignore time-varying risk and resulting changes in the cost of capital in their capital budgeting decisions. If I posit that firms somehow adjust the cost of capital correctly time to time, the cost should negatively predict capital investments. The negative association is because an increase in cost of capital will lower the net present value of investment projects, all other things being equal. I test this theoretical prediction with firm-level data.

Meanwhile, a well-known empirical fact is that corporate investments fluctuate over the business cycles. In particular, firms tend to invest less during recessions and more during expansions in the cycles, over which the equity premium changes counter-cyclically according to the asset pricing studies. Nonetheless, firms that depend entirely on CAPM cannot properly adjust for this time-variation in cost of equity, considering the model's poor performance. Then, does another force, namely financial constraints, lead to the investment pattern? Indeed, Gertler and Gilchrist (1994) and Kashyap, Lamont, and Stein (1994) document that firms tend to be more financially constrained during recessions. Of course, these constraints force firms to reduce investments, thereby generating the procyclical pattern.[†]

[*]Fama and French (1997) and Levi and Welch (2016) find that estimates of the factor loadings and factor risk premiums change depending on both the time of estimation and the length of estimation window, thereby making the estimate of cost of capital imprecise. Consistently, Welch and Goyal (2008) find that existing factor models are almost unable to predict stock returns out-of-sample.

[†]The two forces, time-varying risk and financial constraints, are different in their nature. In a frictionless economy such as in the Modigliani and Miller theorem, financial constraints do not exist, whereas the cost of capital can still

Alternatively, this procyclical investment can result from subjective adjustments by firm managers in their project valuation. In practice, the managers round the cost of capital up or down subjectively in addition to using the CAPM, as found by Graham, Harvey, and Puri (2015) and Jagannathan, Matsa, Meier, and Tarhan (2016). Although the managers are not fully aware of the asset pricing models, these subjective adjustments can possibly lead to investment decisions as if they recognize time-varying risk. Acknowledging these competing explanations for the investment pattern, this study aims to take all relevant forces into account and examine the impact of time-varying risk on capital investments.

A challenge to this analysis is that the cost of equity conditional on a certain date is not observable. To address this circumstance, I propose estimating the cost of equity or equity premium at the firm-level using data on individual equity option prices. Due to their payoff structure, option prices inherently reflect both physical probabilities of future stock prices and investors' risk aversion. Using these information embedded in option prices, I determine the equity premium for each firm. This forward-looking approach requires only current option prices to measure the equity premium. Relying only on up-to-date price information, this measure can better capture variation in cost of capital through time, compared to the existing factor models where estimation results in diluting the recent information with historical data.

To let the option data speak of the equity premium as much as possible, I estimate the premium nonparametrically. The nonparametric approaches that I describe below are highly flexible in specifying the economy, but they are not completely free of any assumption.[‡] Thus, I take two distinct approaches for the estimation. In the first approach, I use each firm's option prices observed in a month and recover the physical probabilities and the stochastic discount factor nonparametrically as in Ross (2015). Through the Euler equation, these two inputs enable me to pin down the firm's equity premium for the month. The second approach, which is proposed by Martin and Wagner (2016), is to measure the lower bound on the equity premium. This approach also utilizes the Euler equation but in a different way and relates the lower bound to the risk-neutral variance of returns that can be obtained from option prices. [§]

change time to time as the business risk fluctuates.

[‡]See Walden (2014) and Borovicka, Hansen, and Scheinkman (2016) for the regularity conditions that the recovery theorem requires.

[§]Martin and Wagner (2016) empirically show that the lower bound forecasts subsequent stock returns and thus the bound is tight.

Using these methods, I estimate monthly equity premium at the firm-level for US non-financial and non-utilities companies constituting S&P 500 index. The estimates exhibit the following properties. First, these two option-implied measures indeed capture the conditional equity premium; in panel regressions using each measure, both of the two measures forecast subsequent returns on equity in excess of the risk-free rate. Second, the two measures display substantial time-variations. For instance, the one-year equity premium measured by the recovery-based approach has the standard deviation of 5.82% compared to the mean of 9.96%. Lastly, these two option-implied measures, although based on different frameworks, move together impressively closely. The regression of the recovery-based measure on the lower-bound measure has the R^2-statistic of 73%. This high R-squared value implies that the two measures capture the same return-relevant information embedded in option prices.

I then investigate whether firms' capital investments respond to the firm-level equity premium. The key result is that the equity premium negatively and significantly forecasts the capital investments. Specifically, both the recovery-based measure and the lower bound measure predict the investment with 1% level of significance, after controlling for other determinants of capital investment noted in the literature, including firm size, market-to-book ratio, profitability, leverage ratio, risk-free rate, and corporate bond yields. Importantly, the negative association is robust to the inclusion of financial constraints. This finding indicates that capital investments reacts negatively to changes in the equity premium, aside from their responses to financial constraints.

Furthermore, to focus on "unlevered" cost of capital that depends only on the fundamental business risk and is independent of a firm's financing structure, I compute the weighted average cost of capital using the equity premium. I then find that an increase in the weighted average cost of capital also drives a decrease in capital investment. Thus, in all of these analyses, it appears that firms adjust for time-varying cost of capital correctly at least in terms of direction of changes in the cost. I interpret this finding as evidence that firm managers' subjective adjustments help to adjust for time-varying risk, which the CAPM fails to capture.

Prior studies on corporate investment, including Lettau and Ludvigson (2002) and Arif and Lee (2014), look at the aggregate-level association between the equity premium and corporate investment and also find the negative association. However, these studies have limitations to speak of the firm-level association. First, extending the aggregate level result into the firm-level is not straightforward, considering the prevalent use of the CAPM in practice that the surveys report. Moreover, the prior studies focus on the aggregate level and they do not control for financial constraints that possibly subsume the predictive power of the equity premium at the firm level. Lastly, they use "proxies" of the equity premium due to absence of a direct measure. I complement these findings by documenting that the firms respond to changes in firm-level equity premium beyond the influence of financial constraints and by establishing the relation using the direct measures of the equity premium.

Having established the negative relationship between the cost of capital and capital investment, I next explore its implications for stock returns. A well-known empirical fact is that firms with larger capital investments tend to have lower subsequent returns on stocks (Titman, Wei, and Xie (2004)). A theoretical explanation for this pattern is time-varying risk; firms invest more when the cost of equity is lower, which is followed by, on average, lower realized returns under the rational expectations hypothesis. To examine whether the investment-return association is attributable to the time-varying risk, I conduct the following test. First, I project the investment onto the space spanned by the equity premium and then regress subsequent stock returns on the projection of the investment.

In the panel regression, I find that the projection of the investment onto the measure of equity premium, either the recover-based measure or the lower bound measure, negatively predicts stock returns in univariate tests. Furthermore, the projection of investment continues to significantly and negatively predict the returns in multivariate tests, in which I control for other relevant variables that Kothari and Shanken (1992) and Lewellen (1999) find explaining time-series of returns. This result supports the hypothesis that the time-varying risk ultimately causes capital investments to negatively predict stock returns. Interestingly, this finding is contrary to the result of Arif and Lee (2014), who use the proxies of the equity premium and conclude that the time-varying risk has no role in explaining the investment-return association.

This study is related to a body of literature on estimating the cost of capital and its relationship to capital investment (Fama and French (1997); Gebhardt, Lee, and Swaminathan (2001); Lettau and Ludvigson (2002); Arif and Lee (2014)). I contribute to the literature by introducing the option-implied measure of cost of capital that is well-suited for capturing time-variation in the cost. This forward-looking measure addresses the concern raised by Fama and French (1997) that estimates of inputs in factor models are imprecise. Meanwhile, Gebhardt et al. (2001) suggests another forward-looking measure that utilizes accounting data. The option-implied measure, in contrast, relies on market price data so avoids noises in accounting report. Using the direct measure of cost of capital, I complement the findings by Lettau and Ludvigson (2002) and Arif and Lee (2014) that the equity premium and corporate investment are negatively associated at aggregate level. Specifically, I document that the negative relationship also holds at the firm-level despite the reported prevalent use of the CAPM and that the relationship is robust to the inclusion of financial constraints.

At the same time, this study provides empirical support for theoretical asset pricing studies, including Campbell and Cochrane (1999) and Bansal and Yaron (2004), by documenting that the estimate of conditional equity premium displays substantial and counter-cyclical variations. It is noteworthy that the equity premium is estimated nonparametrically but exhibit the behavior consistent with these theoretical studies.

Lastly, this study aligns with prior studies that analyze equity option prices and identify implied risk aversion (Ait-Sahalia and Lo (2000); Bliss and Panigirtzoglou (2004)) or physical probability of future stock prices (Backus, Chernov, and Martin (2011)). The main distinction from these prior studies is that I do not assume a functional form for preferences or technology in recovering the option-implied information. Instead, this paper utilizes the nonparametric recovery theorem of Ross (2015) and determine the conditional risk premium. Moreover, this study finds that the option-implied measures of equity premium are reliable. The two measures, the recovery-based measure and Martin and Wagner (2016)'s lower bound measure, move together impressively closely, although they are based on different frameworks.

The remainder of this paper is organized as follows. Section 2 explains the methodology for estimating the firm-level equity premium. Section 3 describes the empirical results on the relations among the risk premium estimates, capital investment, and stock returns. The time-series and cross-sectional properties of the estimated risk premium are described as well. Section 4 concludes.

2 Estimating Firm-Level Equity Premium

This section describes two approaches for estimating the firm-level equity premium implicit in option prices. In both approaches, the objective is to estimate the conditional risk premium for a certain month using individual option prices observed in that month.

The basic setting is as follows. There is no arbitrage, so a positive stochastic discount factor exists. In equilibrium, the Euler equation indicates that the expected return on a stock from date t to T in excess of the risk-free return is

$$E_t(R_T) - R_{f,t} = -R_{f,t}\mathrm{Cov}_t(M_T, R_T) \tag{1}$$

where $R_{f,t}$ is the risk-free return, and M_T is the stochastic discount factor.

The two approaches utilizes the Euler equation in different ways. The recovery-based approach uses a cross-section of option prices with the same expiration dates and employs the recovery therom to determine the joint distribution of the stochastic discount factor and future stock return. Note that through the Euler equation, this joint distribution is able to pin down the equity premium.

On the other hand, the lower-bound approach does not require the joint distribution. Instead, Martin and Wagner (2016) show that the Euler equation relates the lower bound on the equity premium to the risk-neutral variance of stock returns, under certain regularity conditions.[¶] This measure is for the lower bound theoretically, but they show that the bound is empirically tight and forecasts subsequent returns.

Implementing the two methodologies requires prices of European options, whereas most of individual equity options traded on the market are American. To address this circumstance, I compute the price of the European equivalent to each American option using the volatility surface reported by OptionMetrics. Following Carr and Wu (2009) and Martin and Wagner (2016), the reported volatility is entered into the Black-Scholes-Merton formula to generate the European option price.

2.1 The Recovery-Based Measure of Equity Premium

Consider a Markovian economy with a finite number of states. The state is described by the current price of a stock. Let $S_i \in (S_1, \cdots, S_N)$ denote the current stock price on date t.[*]

The first step is to calculate the state price, or the price of Arrow-Debrue security that pays only at a certain future stock price on date T. Breeden and Litzenberger (1978) show that the state

[¶] See Martin and Wagner (2016) and Martin (2017) for details.

[*] Of course, the description of states such as each state's probability and the stochastic discount factor is different for different stocks. One can consider this setup the projection of a more general economic environment onto the space spanned by a stock price. Note that this description of the economy using the filtration of a stock price conveys enough information to determine the equity premium on that stock.

price can be obtained from a cross section of call options having different strike prices and the same time to maturity. That is, for a future price S_j on date T,

$$\text{date-}t \text{ state price of } S_j = R_{f,t} \left. \frac{\partial^2 \text{Call}_T}{\partial X^2} \right|_{X=S_j} \tag{2}$$

where Call_T is the price of call option with the expiration date of T and X is the strike price.

Prior studies suggest different ways to empirically calculate the second derivative of call option price. It would be ideal to use a methodology that does not require any parametric assumption on movement of future stock price for the sake of unbiased estimations. However, the number of observations of individual options is usually not enough large to pursue a perfectly nonparametric estimation. Thus, I use the semiparametric estimator suggested by Ait-Sahalia and Lo (2000). Option prices are determined by the extended Black-Scholes-Merton formula where the implied volatility is a nonparametric function of both the moneyness of the option and the time to maturity. In other words, the call option price is given by

$$\text{Call}_{BSM}\left(F, X, \tau, R_{f,t}, \sigma(X/F, \tau)\right) \tag{3}$$

where F is the forward price of the stock, σ is the implied volatility, $\tau(= T - t)$ is time to maturity, and Call_{BSM} is the Black-Scholes-Merton formula. The function of implied volatility is estimated for each month. To do so, I perform the kernel regression using option prices observed in that month as follows:

$$\widehat{\sigma}\left(X/F, \tau\right) = \frac{\sum_{i=1}^{n} k\left(\frac{X/F - X_i/F_i}{h_{X/F}}\right) k\left(\frac{\tau - \tau_i}{h_\tau}\right) \sigma_i}{\sum_{i=1}^{n} k\left(\frac{X/F - X_i/F_i}{h_{X/F}}\right) k\left(\frac{\tau - \tau_i}{h_\tau}\right)} \tag{4}$$

where i denotes an observation in the month, σ_i is the implied volatility of observation i, $k(z)$ is the Gaussian kernel function such that $k(z) = 1/\sqrt{2\pi} \exp\left(-z^2/2\right)$, and $h_{X/F}$ and h_τ are bandwidth parameters. The bandwidth parameters are chosen to minimize the sum of squared errors of observations as suggested in Hardle (1994). Ait-Sahalia and Lo (2000) show that this semiparametric estimator captures the salient features in the option market, the volatility smile or smirk, which are likely to carry risk-relevant information. As a result, the equity premium that will be recovered from the estimated state price is expected to reflect these option market features.

The above method is designed for the setting where the stock price is a continuous state variable.

I modify the method to apply to the discrete states of stock prices, $(S_1, \cdots, S_N)$ as follows. The state price of S_j on date T when the current price is S_i is

$$Q_{i,j} = R_{f,t} \left. \frac{\partial^2 \text{Call}_{BSM}\left(F_i, X, \tau, R_{f,t}, \widehat{\sigma}\left(X/F_i, \tau\right)\right)}{\partial X^2} \right|_{X=S_j} \left(\frac{S_{j+1} - S_{j-1}}{2} \right). \tag{5}$$

where F_i is the date-T forward price of current stock price S_i. Note that the increment of stock price, $(S_{j+1} - S_{j-1})/2$, is multiplied to obtain the state prices over the discrete states.

Next, I decompose the state prices into the stochastic discount factors and physical probabilities. To do so, I use the Ross (2015)'s recovery theorem. The stochastic discount factor is assumed to be transition independent.† Specifically, the stochastic discount factor when the current stock price is S_i and the future stock price is S_j is

$$M_T = \delta \frac{U_j}{U_i} \tag{6}$$

where δ represents the discount rate and U_i is a positive constant reflecting the marginal utility at state S_i. Then, the state price can be expressed as

$$Q_{i,j} = \delta \frac{U_j}{U_i} F_{i,j} \tag{7}$$

where $F_{i,j}$ is the physical transition probability from state S_i to S_j. In a matrix form, the relation becomes

$$Q = \delta U^{-1} F U \tag{8}$$

where Q is a matrix having $Q_{i,j}$ in row i and column j, F is a matrix having $F_{i,j}$ in row i and column j, and U is a diagonal matrix having U_i in row i and column i.

Let $\mathbb{1}$ denote a column vector of ones having N elements. Because F is the matrix of transition probabilities, $F\mathbb{1} = \mathbb{1}$. Then it follows

$$F\mathbb{1} = \delta^{-1} U Q U^{-1} \mathbb{1} = \mathbb{1} \tag{9}$$

†I recognize the recent critique on the recovery theorem by Borovicka et al. (2016). The critique points out that the assumed transition independence of the stochastic discount factor may not allow the separation of physical probability from a martingale component associated with long-term risk adjustment. To complement this theoretical drawback, I perform the empirical test in a following section to check whether the recovery-based measure correctly captures the conditional equity premium.

and

$$QU^{-1}\mathbb{1} = \delta U^{-1}\mathbb{1}. \tag{10}$$

Denoting $U^{-1}\mathbb{1}$ by a vector $\mathbb{z}$, the result indicates that solving for U^{-1} becomes the problem of finding an eigenvector $\mathbb{z}$ of Q such that $Q\mathbb{z} = \delta\mathbb{z}$. Importantly, Ross (2015) proves that a unique eigenvector exists for the problem, if there is no arbitrage and the matrix Q is irreducible. Let $\widehat{U}$ and $\widehat{\delta}$ denote the obtained solutions from the eigenvector. Plugging these into equation (9), I can also obtain $\widehat{F}$.

Now I can pin down the equity premium using the recovered probability and stochastic discount factor. The expected excess return on a stock of which current price is S_i is

$$E_t(R_T) - R_{f,t} = -R_{f,t}\text{Cov}_t\left(\widehat{\delta}\frac{\widehat{U_T}}{\widehat{U_i}}, \frac{S_T}{S_i}\right) \tag{11}$$

where $S_T \in (S_1, \cdots, S_N)$, U_T is the corresponding marginal utility, and the physical probability $\widehat{F}$ is used to calculate the covariance. So far, the description of economy and the resulting equity premium is specific for one stock. Thus, for stock k, the estimate of date-t equity premium is

$$\text{EP}_{k,t}^{\text{recovery}} = -R_{f,t}\text{Cov}_t\left(\widehat{\delta}\frac{\widehat{U_{k,T}}}{\widehat{U_{k,i}}}, \frac{S_{k,T}}{S_{k,i}}\right) \tag{12}$$

where subscript k is added to highlight that the stochastic discount factors and physical probabilities are recovered from stock k's option prices. In the estimation, $S_{k,i}$ is assumed to be the average of stock k's prices observed in the month.

Note that the entire procedure described above relies only on option prices observed in a month. Thus, performing this estimation month by month will generate a time series of firm-level equity premium.

2.2 The Lower Bound Measure of Equity Premium

The second option-implied measure considered in this study is the lower bound on equity premium, which is introduced by Martin and Wagner (2016). Although this measure is intended to capture the lower bound on equity premium theoretically, they show that the bound is empirically tight and forecasts subsequent return on stocks. In this section, I highlight their main findings. Details of the derivation are provided in the appendix.

Consider a date-T payoff X_T. The payoff's price can be determined either using the stochastic discount factor or the risk-neutral valuation, and the two valuations should lead to the same prices. That is

$$\frac{1}{R_{f,t}} E_t^* \left(X_T \right) = E_t \left(M_T X_T \right) \tag{13}$$

where $E_t^*()$ denotes the expectation under the risk-neutral probability.

First, I discuss on the market portfolio and the lower bound on its expected return, which will be an input for firm-level equity premium. Let $R_{m,t}$ denote the return on the market. It can be proven that using equation (13), the expected return in excess of the risk-free return is

$$E_t \left(R_{m,T} \right) - R_{f,T} = \frac{1}{R_{f,t}} \mathrm{Var}_t^* \left(R_{m,T} \right) - \mathrm{Cov}_t \left(M_T R_{m,T}, R_{m,T} \right). \tag{14}$$

Suppose that the return satisfies the so-called negative correlation condition, $\mathrm{Cov}_t \left(M_T R_{m,T}, R_{m,T} \right) < 0$. Intuitively, this condition requires the stochastic discount factor to be negatively correlated with the return, the property that is likely to hold for the return on market. Martin (2017) shows that this condition actually holds for the market return in most of macro-asset pricing models. Under the negative correlation condition, we obtain the lower bound as follows:

$$E_t \left(R_{m,T} \right) - R_{f,T} \geq \frac{1}{R_{f,t}} \mathrm{Var}_t^* \left(R_{m,T} \right). \tag{15}$$

This lower bound can be computed simply using option prices without any parametric assumption. That is

$$\frac{1}{R_{f,t}} \mathrm{Var}_t^* \left(R_{m,T} \right) = \frac{1}{S_t^2} \left[2 \int_0^\infty \mathrm{Call}_T(K) dK - \frac{1}{R_{f,t}} F_T^2 \right] \tag{16}$$

where $\mathrm{Call}_T(K)$ is the price of call option having the strike price of K and expiration date of T, and F_T is the forward price of stock such that $F_T = E_t^*(S_t)$.

Turning to individual stocks, the negative correlation condition, a key assumption in deriving the lower bound on the market equity premium, may not hold at the firm-level. For example, a stock on a company of which business is largely associated with a hedge asset, such as gold, may perform well when the stochastic discount factor is large. Hence, one cannot simply extend the result for the market equity premium to characterize a firm-level equity premium.

Instead, Martin and Wagner (2016) consider a hypothetical portfolio that generates the maximum expected log return using tradable assets. They prove that the return on that portfolio, referred to as growth-optimal return, is the same as the inverse of the stochastic discount factor. Thus, using the Euler equation for each asset, returns on any pair of two assets can be related to each other through the growth-optimal return.

Using the relation, return on stock k, $R_{k,T}$, can be expressed as follows:

$$E_t\left(R_{k,T}\right) - R_{f,t} = \alpha_k + E_t\left(R_{m,T}\right) - R_{f,t} + \frac{R_{f,t}}{2}\left[\text{Var}_t^*\left(\frac{R_{k,T}}{R_{f,t}}\right) - \sum_k w_{k,t}\text{Var}_t^*\left(\frac{R_{k,T}}{R_{f,t}}\right)\right] \tag{17}$$

where α_k is a constant, $w_{k,t}$ is the weight of stock k in the stock market index. The risk-neutral variance, $\text{Var}_t^*\left(R_{k,T}/R_{f,t}\right)$, can be computed using individual equity option prices as in equation (16). Finally, plugging the lower bound on market equity premium into equation (17), the lower bound on a firm's equity premium is [‡]

$$\text{EP}_{k,t}^{\text{lower bound}} = \alpha_k + \frac{1}{R_{f,t}}\text{Var}_t^*\left(R_{m,T}\right) + \frac{R_{f,t}}{2}\left[\text{Var}_t^*\left(\frac{R_{k,T}}{R_{f,t}}\right) - \sum_k w_{k,t}\text{Var}_t^*\left(\frac{R_{k,T}}{R_{f,t}}\right)\right]. \tag{18}$$

Using option prices observed in a certain month, I can compute the risk-neutral variances and the resulting lower bound on equity premium for that month. Thus, in a similar way to the recovery-based equity premium, I move the data window month by month and obtain the time-series of the lower bound.

3 Empirical Results

3.1 Data

I focus the analysis on US non-financial and non-utilities companies that are included in the S&P 500 at least once during years 1996 to 2014. This focus is to ensure that each company has a enough large number of observations for individual equity option prices, which is required to estimate the

[‡] Martin and Wagner (2016) show that the constant α_k is empirically insignificant. Following the result, I set α_k to be zero for all stocks.

firm-level equity premium.

Data on individual equity options are obtained from OptionMetrics for the period January 1996 through August 2014. As described previously, the estimation utilizes the prices of European call options having different strike prices. Among the call options, in-the-money calls tend to be illiquid, thereby possibly having the price deviating from the fair value. Thus, for an in-the-money call, I instead use the price of the put option with the same strike price and maturity and compute the price of the call via the put-call parity. In addition, in estimating a firm's equity premium for a specific month, I require the firm to have at least 30 option price observations in that month. Applying the filter results in 651 firms having on average 110 monthly estimates of the recovery-based equity premium and 776 firms having on average 105 monthly estimates of the lower bound.

For these companies in the sample, I collect quarterly financial statements from Compustat. Firm-level variables are measured in standard ways in literature. A firm's investment-capital ratio in quarter t is defined as capital expenditures (CAPXY) in quarter t divided by property, plant and equipment (PPENT) in quarter $t-1$. Book leverage ratio ($\text{LEV}_{i,t}$) is the sum of debt in current liabilities (DLCQ) and long-term debt (DLTTQ) divided by total assets (ATQ). Profitability of a firm is measured by return on assets ($\text{ROA}_{i,t}$), which is current net income (NIQ) divided by previous total assets (ATQ). Firm size ($\text{SIZE}_{i,t}$) is defined as the natural log of total assets. The level of cash flow ($\text{CF}_{i,t}$) is income before extraordinary items (IBQ) plus depreciation and amortization (DPQ). Cash holdings ($\text{Cash}_{i,t}$) are cash and short-term investments (CHEQ) and dividends ($\text{Dividends}_{i,t}$) are cash dividends (DVY). Sales growth ($\text{Sales Growth}_{i,t}$) is the rate of growth in sales (SALEQ) from $t-1$ to t. Industry sales growth is the average of the growth rates for companies that belong to three-digit industry. The ratio of the market-to-book value ($Q_{i,t}$) is measured by (total assets (ATQ) + market value of equity (MKVALTQ) - book value of equity (CEQQ)) divided by total assets (ATQ).

To determine firms' overall cost of capital, the cost of debt is also needed. I obtain yields on corporate bonds from Trade Reporting and Compliance Engine (TRACE). The observed yields in quarter t for firm i's bonds are aggregated to generate the weighted average of the yields ($\text{YIELD}_{i,t}$), where the weight is the par value of each bond. The risk-free returns are 10-year treasury constant maturity rates from Federal Reserve Economic Data.

In addition, I also include the traditional measure of the cost of equity, by calculating the

CAPM-beta of each firm's stock. The quarter-t conditional beta is estimated by regressing daily excess returns of individual stock in the past five years on the market excess return in the same period.

Finally, my analysis needs a measure of financial constraints. I use three different measures suggested in literature - a level of cash flow, the index of Kaplan and Zingales (1997) (KZ index), and the index of Whited and Wu (2006) (WW index). Using the level of cash flow is based on the view of Fazzari, Hubbard, and Petersen (1988) that a firm with larger internal cash flow is less likely to suffer from a high cost of external financing. To standardize the measure, I compute a Z-score of cash flow using historical observations of each firm's cash flow. Specifically, to obtain the firm i's Z-score of cash flow in quarter t, I compute the standard deviation $\sigma_{i,t}(\mathrm{CF})$ and the average $m_{i,t}(\mathrm{CF})$ of past five-year observations of quarterly cash flows and define the Z-score as

$$\text{Z-score of } \mathrm{CF}_{i,t} = \frac{\mathrm{CF}_{i,t} - m_{i,t}(\mathrm{CF})}{\sigma_{i,t}(\mathrm{CF})} \tag{19}$$

where $\mathrm{CF}_{i,t}$ is cash flow in quarter t. Next, as alternative measures of financial constraints, I compute the KZ index and the WW index by employing the estimation results reported in Lamont, Polk, and Saa-Requejo (2001) and Whited and Wu (2006), respectively.[§] Note that these two indexes indicate the degree of financial constraints in the opposite way of the cash flow measure. A higher Z-score of cash flow indicates that a firm is less constrained, whereas a higher KZ index or WW index shows that a firm is more constrained.

Table 1 presents descriptive statistics of these variables.

[§]In the original studies, these two indexes are constructed using annual variables. To generate consistent indexes using quarterly variables in this study, I multiply the coefficients for the flow variables by four. The resulting KZ index is

$$\begin{aligned}\text{KZ index} = &-(4)(1.002) \times \frac{\mathrm{CF}}{\text{Net Fixed Assets}} + 0.283 \times Q + 3.139 \times \mathrm{LEV} - (4)(39.368) \times \frac{\text{Dividends}}{\text{Net Fixed Assets}} \\ &- 1.315 \times \frac{\text{Cash}}{\text{Net Fixed Assets}}.\end{aligned}$$

The WW index is

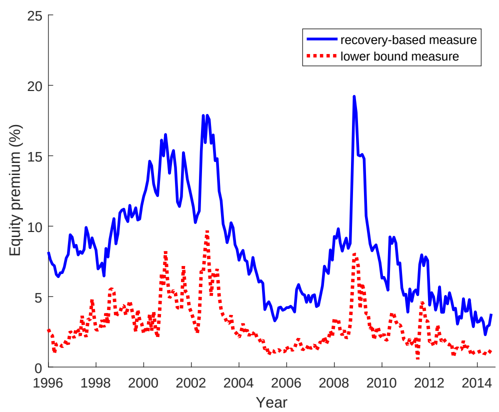
$$\begin{aligned}\text{WW index} = &-(4)(0.091) \times \frac{\mathrm{CF}}{\text{Total Assets}} - 0.062 \times \mathbb{1}_{\text{Dividneds}>0} + 0.021 \times \mathrm{LEV} - 0.044 \times \mathrm{SIZE} \\ &+ (4)(0.102) \times \text{Industry Sales Growth} - (4)(0.035) \times \text{Sales growth}\end{aligned}$$

where $\mathbb{1}_{\text{Dividends}>0}$ is an indicator variable having the value of one if dividends are positive and zero otherwise.

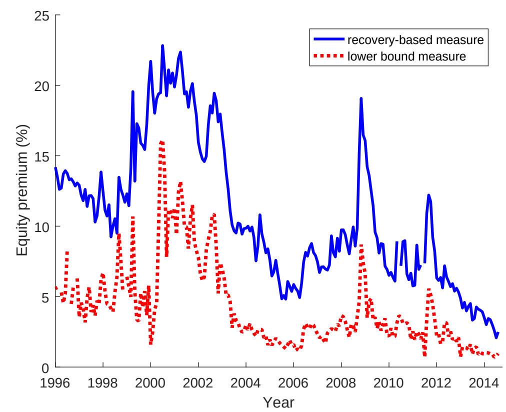
Figure 1: Examples of Estimated Equity Premium

This figure presents the monthly time-series of the estimated equity premium for selected firms. Selected are Intel and Qualcomm in industry of semiconductors and related devices and Exxon Mobil and Chevron in industry of petroleum petroleum products. The equity premium is for six-month horizon.

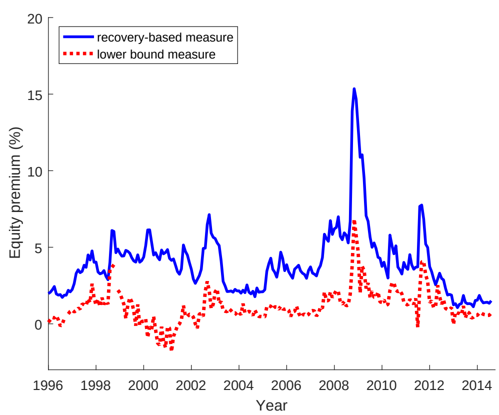
(a) Intel



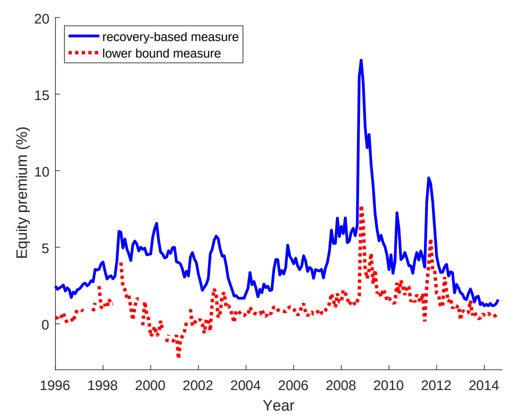
(b) Qualcomm



(c) Exxon Mobil



(d) Chevron



3.2 A Look at the Estimates of Firm-Level Equity Premium

This study takes a new approach to estimating the cost of equity, so it is worth discussing general features of the estimated cost. Figure 1 depicts the monthly time-series of the equity premium for selected firms: Intel and Qualcomm in the semiconductors and related devices industry and Exxon Mobil and Chevron in the petroleum and petroleum product industry. The plotted equity premiums are for six-month horizon.

All four of the companies display substantial time-variations in the equity premium. Moreover, the equity premium sometimes changes rapidly, in particular during 2007-08 financial crisis. Thus, if firms simply depend on historical data and the factor models to determine the cost of capital, they would not be able to adjust to these rapid changes in the cost. Note that these time-variations are obtained entirely from the data and not from a model assumption. This obtained feature provides empirical support for the time-varying risk that the theoretical asset pricing models are based on.

In addition, the estimated equity premium has a cross-sectional property that one would expect for the cost of capital. The equity premiums for companies that belong to the same industry are more highly correlated than those for companies from different industries. As examples of intra-industry pairs, the correlation coefficient between the recovery-based measures is 0.95 for Exxon Mobil and Chevron and 0.72 for Intel and Qualcomm. On the other hand, inter-industry pairs have lower correlation coefficients, 0.35 for Intel and Exxon Mobil and -0.15 for Qualcomm and Chevron.

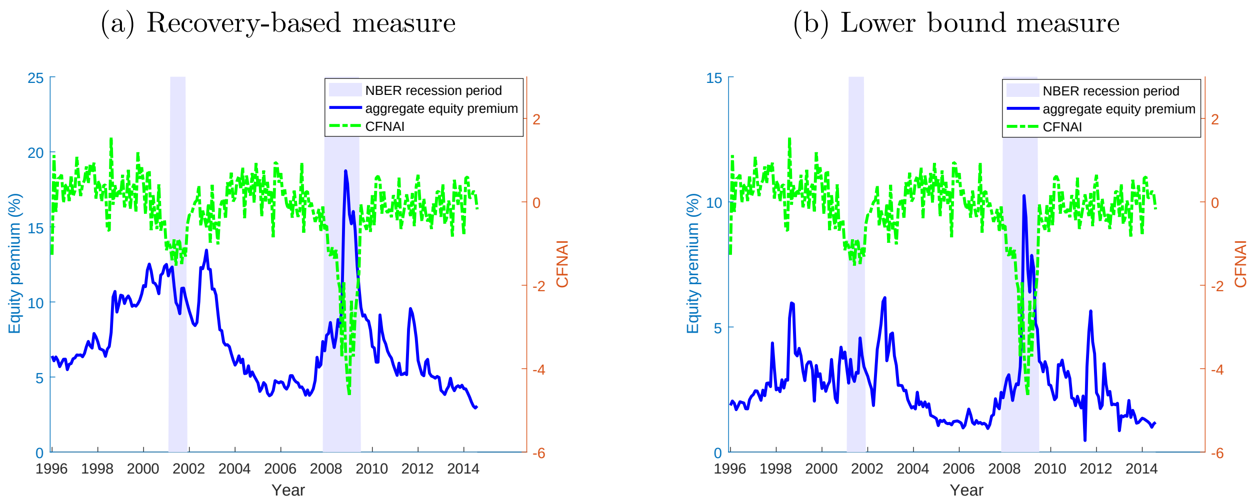
Comparing the two measures of the equity premium, the lower bound measure is consistently lower than the recovery-based measure. This is not surprising, however, because theoretically the lower bound measure is intended to capture the lower bound on the equity premium, whereas the recovery-based measure is to be an unbiased estimator of the premium. Despite the difference in the level, the two measures move together closely. For example, the correlation coefficient between the two measures is 0.89 for Intel. The compatibility between these two measures will be discussed more in the next section.

Although it is not a main focus of this study, I also investigate the behavior of the market-wide equity premium using the firm-level estimates. As a rough approximation of the market, I sample 73 companies that have more than 200 estimates of the monthly equity premium in the sample period. I then compute the equally weighted average of the firm-level estimates across the companies for each month. Figure 2 plots the resulting aggregate equity premium. The left column shows the aggregate estimates from the recovery-based measure and the right column shows the estimates from the lower bound measure. I find that the aggregated equity premium from the two measures is counter-cyclical; the aggregated premium is notably high during the NBER recessions of 2008 and 2001, whereas it decreases during the boom period of mid-2000.

To further diagnose the counter-cyclicality, I examine the association between the aggregate equity premium and the Chicago Federal Reserve National Activity Index (CFNAI), an indicator of economic growth. The two series are contemporaneously negatively correlated with the correlation

Figure 2: Aggregate Equity Premium

This figure presents the monthly time-series of the cross-sectional average of firm-level equity premiums of 73 companies. In addition, the Chicago Federal Reserve National Activity Index (CFNAI) is plotted for the same period. The shaded areas depict the NBER recession periods.



coefficient of -0.53 for the recovery-based measure and -0.55 for the lower bound measure. This reconfirms a rise in equity premium during economic contraction.

To summarize, I document that the estimates of firm-level equity premium indeed display substantial time-variations. Next, I look into whether these estimates predicts subsequent stock return.

3.3 The Firm-Level Equity Premium and Realized Stock Returns

In this section, I examine the relationship between the option-implied measures of equity premium and subsequently realized stock returns. This analysis can be considered testing the empirical validity of the option-implied measures. If the measures indeed capture the firm-level equity premium, they should forecast the realized excess returns on stock. Once I empirically establish this relationship, I can regard the option-implied measure as representing the conditional equity premium and use the measure in the following analysis.

To test whether the option-implied measures capture the equity premium, I perform a pooled regression,

$$R^e_{i,T} = \alpha + \beta \text{EP}_{i,t} + \epsilon_{i,T} \tag{20}$$

where $R^e_{i,T}$ is the realized return on stock from date t to T in excess of the risk-free rate, $\text{EP}_{i,t} \in \left(\text{EP}^{\text{recovery}}_{i,t}, \text{EP}^{\text{lower bound}}_{i,t}\right)$ is the date-t option-implied equity premium over the period from t to T. If the theory underlying the option-implied measures holds perfectly, I expect to find that $\alpha = 0$ and $\beta = 1$.

Panel A of Table 2 reports the regression results for return horizons of six months and one year. The main finding is that both the recovery-based measure and the lower bound measure of equity premium predict subsequent returns at 1% level of significance. The t-statistics of the recovery-based measure is 7.51 for six-month horizon and 9.95 for one-year horizon. Similarly, the t-statistics of the lower-bound measure is 9.27 for six-month horizon and 11.79 for one-year horizon. Furthermore, the coefficients on the option-implied measures are close to one, the value the theory predicts. The coefficients on the recovery-based measure (lower-bound measure) are 0.451 (0.837) for six-month horizon and 1.016 (0.917) for one-year horizon.

To test the null hypothesis that $\alpha = 0$ and $\beta = 1$, I conduct the Wald test. The null hypothesis is rejected for all specifications. However, a weaker hypothesis that $\beta = 1$ is accepted in most specifications with $p-$values between 0.07 and 0.88 except for the recovery-based measure of six-month equity premium. These results tell that the option-implied measures capture equity premium changes correctly in both magnitude and direction, although a bias exists in predicting the level of the premium. In the main analysis in the following section, I will focus on within-firm changes of equity premium and firms' responses, so having a non-zero constant in this return regression would not be problematic.

The predictive power of the option-implied measures may come from both cross-sectional variation and time-series variation in stock return that the measures explain. To focus on the measures' ability to predict the time-series variations, I perform the following panel regression

$$R^e_{i,T} = \alpha_i + \beta \text{EP}_{i,t} + \epsilon_{i,T} \tag{21}$$

where the firm dummy α_i is included. The regression results are reported in panel B of Table 2.

The option-implied equity premiums continue to predict subsequent stock returns with even larger t-statistics, after the firm fixed effect is accounted for. This significance confirms that the option-implied estimates detect well within-firm variations across time in equity premium. This property of the option-implied measure is crucial for the main analysis, where I will investigate the time-series relationship between equity premium and capital investments.

Focusing on the recovery-based measure, a theoretical concern is raised recently by Borovicka et al. (2016). They point out that the assumed structure in the recovery theorem might lead to a misspecification; the physical probability might not be separated from a martingale component associated with long-term risk adjustment. The above empirical tests show that the recovery-based measure indeed tends to slightly underestimate the level of equity premium. Despite the small bias, it appears that the measure still significantly predicts within-firm variations of the premium with the reasonable coefficients close to one. Therefore, I take the measure as a suitable indicator of the equity premium for this study.

Having a new predictor for stock returns, one may wonder how the option-implied measure would compete with existing firm characteristics that the literature find to predict stock returns. Based on the idea, I add the relevant variables suggested by Kothari and Shanken (1992) and Lewellen (1999) to the panel regression of specification (21). The firm characteristics include the market-to-book ratio, firm size, leverage ratio, lagged realized returns, and investment-capital ratio. The regression results in Table 3 show that the two option-implied measures remain significant in predicting stock returns. Furthermore, the coefficient estimates on the measures change only slightly after the inclusion of other predictors. This suggests that most of return-relevant information that the option-implied measures convey is not explained by existing firm characteristics.

Lastly, I investigate how much compatible the two option-implied measures, recovery-based measure and lower-bound measure, are to each other. Despite different theoretical frameworks underlying the two measures, two estimates should be close to each other if they contain the same return-relevant information. To check the compatibility, I run the following contemporaneous regression

$$\mathrm{EP}_{i,t}^{\text{recovery}} = \alpha + \beta \mathrm{EP}_{i,t}^{\text{lower bound}} + \epsilon_{i,T} \tag{22}$$

and also run the panel regression that includes the firm fixed effect.

The regression results are presented in Table 4. First, I find that the coefficients on the lower-bound measure are significantly positive and moreover close to one in both the pooled and panel regressions. The coefficients are 1.281 (1.120) for six-month horizon and 0.684 (0.594) for one year horizon in the pooled regression (in the panel regression). This result indicates that the equity premium measured by the two approaches move together closely not only in the same direction but also by a similar magnitude. Focusing on the pooled regression, the intercept is positive, meaning that the recovery-based estimate of equity premium is on average larger than the lower-bound estimate. This result is not surprising, however, because the lower-bound measure is theoretically intended to capture the lower bound on equity premium, whereas the recovery-based measure is to be the unbiased estimator of equity premium. More importantly, the R^2-statistics are impressively large. For example, the statistics for six-month horizon is 65.99% in the pooled regression and 72.63% in the panel regression. Having the property that most of variations in one measure is explained by the other, I argue that these two option-implied measure are almost empirically equivalent in capturing time-variation in equity premium.

To summarize, I have empirically established that the two option-implied measures indeed capture the conditional equity premium at the firm level. From now on, I use the two measures to study the relationship between corporate investment and the cost of capital.

3.4 The Cost of Capital and Capital Investment

I now examine whether firms take the fluctuating equity premium into account in their decisions on capital investment. A rise in the equity premium increases a firm's cost of capital. Thus, if firms correctly adjust for time-varying risk in determining cost of capital, I expect to find a negative association between the investment and the equity premium, all other things being equal.

To test this hypothesis, I run the following panel regression:

$$\text{INVEST}_{i,t} = \beta \text{EP}_{i,t-1} + \gamma X_{i,t-1} + \alpha_i + \epsilon_{i,t+1} \tag{23}$$

where $\text{INVEST}_{i,t}$ is firm i's investment-capital ratio in quarter t, $\text{EP}_{i,t-1}$ is the quarter $t-1$ estimate of the equity premium for six-month horizon[¶], either $\text{EP}^{\text{recovery}}_{i,t-1}$ or $\text{EP}^{\text{lower bound}}_{i,t-1}$, and $X_{i,t-1}$ is control variables. The controls include profitability ($\text{ROA}_{i,t-1}$), risk-free return (r^f_{t-1}), the firm size measured by the log of book value of total assets ($\text{SIZE}_{i,t-1}$), the market-to-book ratio ($Q_{i,t-1}$), leverage ratio ($\text{LEV}_{i,t-1}$), the bond yield ($\text{YIELD}_{i,t-1}$), and a measure of financial constraints.

[¶]I calculate the average of the monthly estimates to obtain the quarterly equity premium.

Table 5 presents the regression results. The main focus of this analysis is the coefficients of the option-implied equity premium. Specification (1) and (2) show that the equity premium, whether it is measured by the recovery-based approach or the lower bound approach, negatively predicts capital investments. This negative association is statistically significant at 1% level with t-statistics of -3.88 for the recovery-based measure and -4.31 for the lower bound measure. This finding indicates that firms reduce (increase) their capital investment when the equity premium increases (decreases). Hence, it appears that firms adjust their investment decisions correctly to fluctuations in equity premiums, at least in terms of direction of the fluctuations.

Another important determinant of capital investment is financial constraints. Controlling for financial constraints is particularly critical in assessing the relationship between the investment and equity premium. This is because time-varying risk and financial constraints are two distinct forces, but they are expected to cause similar fluctuations in the investments over business cycles. According to studies by Gertler and Gilchrist (1994) and Kashyap et al. (1994), firms tend to be more financially constrained during recessions. Considering this nature and its impact on firm investments, the force of financial constraints has a potential to subsume the predictive ability of equity premium. Moreover, the CAPM, the de facto standard framework in practice for determining the cost of capital, performs only poorly in capturing time-varying risk. Thus, it is reasonable to cast doubt on the impact of time-varying risk on firms' investments and to postulate that the procyclical investment is only driven by financial constraints. Having this alternative explanation in mind, I control for various measures of financial constraints in specifications (3) through (8).

The result is that financially constrained firms tend to invest less, as the theory predicts. In particular, the Z-score of cash flows positively predicts the investments in specifications (3) and (4), and both the KZ-index in (5) and (6) and WW-index in (7) and (8) negatively predicts, although the statistical significance varies across specifications. Turning to the role of the equity premium, the key finding is that the equity premium maintains the explanatory power at 1% level of significance for all specifications. In other words, capital investments respond negatively to changes in the equity premium, aside from their responses to financial constraints. I interpret these findings of specifications (1) through (8) as evidence that firm managers correctly adjust for time-varying risk in spite of the CAPM's near inability to capture it.

Next, in specifications (9) and (10), I conduct a horse race between the option-implied equity premium and the traditional CAPM-based equity premium. The idea behind this analysis is that if firm managers rely entirely on the CAPM in determining the cost of capital, the CAPM-based equity premium would predict capital investment and the option-implied equity premium might lose the predictive power. The result is that the option-implied equity premium still strongly predicts the investment. On the other hand, the coefficient on CAPM-based equity premium is insignificant or positive. Interestingly, the positive association is the opposite of the theoretical prediction that a rise in the cost of capital will result in a decrease in capital investment. This poor and inconsistent performance of the CAPM-based equity premium suggests that firm managers do not depend entirely on the CAPM in practice. Instead, they appear to make substantial adjustments to the cost of capital beyond what the CAPM suggests, and these adjustments help them to reflect time-varying risk in project valuations.

To make a clear distinction between the impact of financial constraints and the impact of fundamental business risk, it is better to have the cost of capital that is independent of firms' financing choices. Motivated by this idea, I calculate "unlevered" cost of capital, by computing the weighted average cost of capital (WACC) for each firm. The WACC is obtained as follows:

$$\mathrm{WACC}_{i,t} = (1 - \mathrm{LEV}_{i,t}) \cdot (\mathrm{EP}_{i,t} + R_{f,t}) + \mathrm{LEV}_{i,t} \cdot \mathrm{YIELD}_{i,t} \tag{24}$$

where $\mathrm{LEV}_{i,t}$ is the leverage ratio, $\mathrm{YIELD}_{i,t}$ is the average of corporate bond yields, and $R_{f,t}$ is the risk-free rate. In the frictionless world, the WACC is solely determined by the business risk and independent of the firm's capital structure. Hence, the time-series of WACC enables me to focus on variations in the business risk and assess their impact on capital investment.

Table 6 presents the results of regressing capital investment on the WACC and other controls. I find that the WACC negatively predicts the investment at 5% or lower level of significance in all specifications. Importantly, this finding confirms that time-variations in business risk is properly taken into account in managers' determining the cost of capital. Therefore, the equity premium's negative prediction is not entirely driven by the force of financial constraints. Compared to results in Table 5, however, I find that the statistical significance of the WACC is slightly lower than that of the equity premium, while the indicator of financial constraints gain additional significance. This indicates that the impact of equity premium on the investment reflects both forces of time-varying business risk and financial constraints.

To summarize, the firm-level analysis establishes that corporate investments respond negatively to fluctuations in both equity premiums and the overall cost of capital. The negative associations reveals that firm managers correctly adjust for time-varying risk despite the CAPM's poor performance in capturing it.

3.5 Reassessing the Relation between Capital Investment and Subsequent Stock Returns

In this section, I explore the implications of the main finding, the negative prediction of time-varying risk for capital investment, for the relation between the investment and subsequent stock returns. A well-known empirical fact is that firms with larger capital investment tend to have lower subsequent returns on their stocks (Titman et al. (2004)). The firm panel of this study also exhibits this pattern. Table 3 in the previous section shows that the investment-capital ratio negatively predicts subsequent stock returns with the t-statistic of -3.01 for six-month horizon and -2.72 for 12-month horizon.

A theoretical explanation for this negative investment-return association is time-varying risk; firms invest more when equity premium is low, followed by, on average, lower realized returns under the rational expectations hypothesis. However, testing this hypothesis has been challenging since equity premiums are not observable. At best, prior study by Arif and Lee (2014) tries to use "proxies" for equity premium and finds no role of time-varying risk in explaining the investment-return association.

Now, having the direct measures of firm-level equity premium, I can reexamine whether time-varying risk contributes to generating the negative investment-return association. To do so, I take the following steps. First, I isolate the changes in capital investment that are attributable to time-varying risk apart from the changes in the investment driven by other forces. Specifically, in the regression of the investment specified as in equation (23), I obtain the projection of the investment onto the equity premium and also calculate the residual in the investment after the projection. Second, stock returns are regressed on the projection and the residual of investment as well as other control variables. If the time-varying risk is the channel through which investment forecasts returns, I expect to see a negative link between stock returns and the projection of investment.

Table 7 presents the regression results. $\mathbb{P}_{\mathrm{EP}_{i,t-1}}\mathrm{INVEST}_{i,t}$ denotes the projection of investment onto the equity premium and $\mathbb{R}_{\mathrm{EP}_{i,t-1}}\mathrm{INVEST}_{i,t}$ is the residual after the projection. The investment is projected either the recovery-based measure, $\mathrm{EP}_{i,t-1}^{\mathrm{recovery}}$, or the lower-bound measure, $\mathrm{EP}_{i,t-1}^{\mathrm{lower\ bound}}$. Specifications (1) and (2) include only the projection and residual of investment as explanatory variables. The result is that subsequent returns are negatively associated with the projection of investment onto either of the two measures of equity premium. The t-statistic for the projection onto the recovery-based equity premium is -6.90, and that onto the lower bound on equity premium is -10.20. Controlling for other relevant variables in specifications (3) and (4), the projections of investment continue to be significant predictors. This consistent negative association indicates that time-varying risk ultimately accounts for the investment-return association, at least to some extent. Interestingly, this result is contrary to the finding by Arif and Lee (2014) who use proxies of equity premium and conclude that the investment-return association comes entirely from irrational sentiment.

On the other hand, the residual of investment after the projection significantly predicts returns too. In other words, changes in investment orthogonal to time-varying risk also have a predictive power for returns. I conjecture that the residual component in investment contains information regarding future cash flows or investors' sentiment, which also influence stock returns beside the discount rate.

Overall, using the direct measures of equity premium, I document evidence supporting the hypothesis that firm investments' response to time-varying risk leads to the established fact of lower stock returns after larger capital investment. Time-varying risk, however, is not the only impetus of the investment-return relation, and other determinants of capital investment - for example, news on future cash flows - also play a role in generating the association.

4 Conclusion

Capital budgeting decisions are crucial to firms' value creation. Among many aspects to consider in capital budgeting, this study focuses on characterizing the cost of capital. Recent studies in asset pricing reveal that the equity premium changes substantially time to time. If a firm ignores the time-variations in determining the cost of capital, value implications can be substantial; such a firm would incur a 15% loss of total firm value according to Kim and Routledge (2015). Motivated by the sizable value loss that the incorrect cost of capital would cause, I empirically investigate whether firms take time-varying risk into account in their investment decisions.

To conduct this empirical test requires a measure of a conditional cost of equity. I take new approach to estimating firm-level equity premium, using individual equity option prices. In particular, I nonparametrically estimate the equity premium using two distinct frameworks, which extract the option-implied information in different ways. I found that the two estimates of equity premium move together impressively closely, although they are based on different approaches. Furthermore, the option-implied equity premium forecasts subsequent stock returns and display counter-cyclical variations, consistent with the asset pricing models.

The main analysis reveals that the equity premium, measured by either recovery-based approach or lower bound approach, negatively predicts capital investment. The negative association is robust to the inclusion of financial constraints. Furthermore, capital investments also respond to negatively to the unlevered cost of capital that is independent of financing choices. These findings suggests that managers correctly adjust the cost of capital to fluctuations in the business risk, in spite of the CAPM's near inability to capture time-varying risk.

In addition, I explore the implications for stock returns of the negative link between the equity premium and corporate investment. The evidence supports the hypothesis that the time-varying risk causes larger capital investments to predict lower subsequent stock returns. However, I find that the changes in investment driven by forces other than time-varying risk also negatively predict the returns. The result indicates that time-varying risk is not the only source generating the investment-return association.

This study is far from providing a complete picture on the actual process of determining cost of capital in practice. Beyond the qualitative link between the cost of capital and the investment in time-series, it will be also interesting to look into quantitatively how the option-implied estimates compares to the actual discount rates that firms use. I leave an exploration of this question to future research.

References

Ait-Sahalia, Y., Lo, A. W., 2000. Nonparametric risk management and implied risk aversion. Journal of Econometrics 94, 9–51.

Arif, S., Lee, C. M., 2014. Aggregate investment and investor sentiment. Review of Financial Studies 27, 3241–3279.

Backus, D., Chernov, M., Martin, I., 2011. Disasters implied by equity index options. Journal of Finance 66, 1969–2012.

Bansal, R., Yaron, A., 2004. Risk for the long run: a potential resolution of asseet pricing puzzles. Journal of Finance 59, 1481–1509.

Bliss, R. R., Panigirtzoglou, N., 2004. Option-implied risk aversion estimates. Journal of Finance 59, 407–446.

Borovicka, J., Hansen, L. P., Scheinkman, J. A., 2016. Misspecified recovery. Journal of Finance 71, 2493–2544.

Breeden, D. T., Litzenberger, R. H., 1978. Prices of state-contingent claims implicit in option prices. Journal of Business 51, 621–651.

Campbell, J. Y., Cochrane, J. H., 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107, 205–251.

Carr, P., Wu, L., 2009. Variance risk premiums. Review of Financial Studies 22, 1311–1341.

Cochrane, J. H., 2011. Presidentail address: Discount rates. Journal of Finance 66, 1047–1108.

Fama, E. F., French, K. R., 1997. Industry costs of equity. Journal of Financial Economics 43, 153–193.

Fazzari, S. R., Hubbard, G., Petersen, B., 1988. Financing constraints and corporate investment. Brookings Papers on Economic Activity pp. 141–195.

Gebhardt, W. R., Lee, C. M. C., Swaminathan, B., 2001. Toward an implied cost of capital. Journal of Accounting Research 39, 135–176.

Gertler, M., Gilchrist, S., 1994. Monetary policy, business cycles, and the behavior of small manufacturing firms. Quarterly Journal of Economics 109, 309–340.

Graham, J. R., Harvey, C. R., 2001. The theory and practice of corporate finance: evidence from the field. Journal of Financial Economics 60, 187–243.

Graham, J. R., Harvey, C. R., Puri, M., 2015. Capital allocation and delegation of decision-making authority within firms. Journal of Financial Economics 115, 449–470.

Hardle, W., 1994. Applied Nonparametric Regression. Cambridge University Press.

Jagannathan, R., Matsa, D. A., Meier, I., Tarhan, V., 2016. Why do firms use high discount rates? Journalf of Financial Economics 120, 445–463.

Kaplan, S. N., Zingales, L., 1997. Do investment-cash flow sensitivities provide useful measures of financing constraints? Quaterly Journal of Economics 112, 169–215.

Kashyap, A. K., Lamont, O. A., Stein, J. C., 1994. Credit conditions and the cyclical behavior of inventories. Quarterly Journal of Economics 109, 565–592.

Kim, Y., Routledge, B. R., 2015. Does macro-asset pricing matter for corporate finance? Working Paper .

Kothari, S., Shanken, J., 1992. Stock return variation and expected dividends. Journal of Financial Economics 31, 177–210.

Lamont, O., Polk, C., Saa-Requejo, J., 2001. Financial constraints and stock returns. Review of Financial Studies 14, 529–554.

Lettau, M., Ludvigson, S., 2002. Time-varying risk premia and the cost of capital: An alternative implication of the q theory of investment. Journal of Moneytary Economics 49, 31–66.

Levi, Y., Welch, I., 2016. Asssing cost-of-capital inputs. Working Paper .

Lewellen, J., 1999. The time-series relations among expected return, risk, and book-to-market. Journal of Financial Economics 54, 5–43.

Martin, I., 2017. What is the expected return on the market? Quarterly Journal of Economics 132, 367–433.

Martin, I., Wagner, C., 2016. What is the expected return on a stock? Working Paper .

Ross, S., 2015. The recovery theorem. The Journal of Finance 70, 615–648.

Titman, S., Wei, K. J., Xie, F., 2004. Capital iinvestment and stock returns. Journal of Financial and Quantitative Analysis 39, 677–700.

Walden, J., 2014. Recovery with unbounded difussion processes. Working Paper .

Welch, I., Goyal, A., 2008. A comprehensive look at the empirical performance of equity premium prediction. Review of Financial Studies 21, 1455–1508.

Whited, T. M., Wu, G., 2006. Financial constraints risk. Review of Financial Studies 19, 531–559.

Table 1: Summary Statistics

This table presents descriptive statistics of firm-level variables. The statistics are calculated from annualized variables.

Summary Statistics					
Variable	Mean	Std.Dev.	25%	50%	75%
$\text{EP}_{i,t}^{\text{recovery}}$ (%)	9.996	5.829	5.830	8.763	12.831
$\text{EP}_{i,t}^{\text{lower bound}}$ (%)	6.065	7.034	2.003	3.945	7.426
$\text{EP}_{i,t}^{\text{CAPM}}$ (%)	10.323	12.546	1.082	7.508	17.926
INVEST_t	0.260	0.377	0.123	0.194	0.303
SIZE_t	8.456	1.463	7.543	8.412	9.414
ROA_t (%)	5.794	84.448	2.440	6.256	10.607
LEV_t	0.246	0.122	0.232	0.343	0.321
Q_t	2.346	2.426	1.334	1.781	2.588
YIELD_t (%)	5.017	3.129	3.131	4.623	5.951
Z-score of CF_t	0.475	1.794	-0.247	0.322	0.992
KZ index_t	-1.340	4.505	-1.543	0.122	0.857
WW index_t	-0.409	0.088	-0.469	-0.412	-0.353

Table 2: The Option-Implied Equity Premium and Subsequent Stock Returns

This table presents regressions of realized stock returns on the option-implied equity premium. The dependent variable is realized excess return for the following six month or one year from time t. In each regression, the option-implied equity premium is either $\mathrm{EP}_{i,t}^{\text{recovery}}$ or $\mathrm{EP}_{i,t}^{\text{lower bound}}$ for the corresponding horizon. Panel A reports the result of the pooled regression for the specification

$$R_{i,T}^{e} = \alpha + \beta \mathrm{EP}_{i,t} + \epsilon_{i,T}.$$

Panel B reports the result of the panel regression including firm fixed effects as follows:

$$R_{i,T}^{e} = \alpha_i + \beta \mathrm{EP}_{i,t} + \epsilon_{i,T}.$$

The t-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Panel A. Pooled Regression

Dependent variable:	$R_{i,T}^{e}$			
Return Horizon:	6 months	6 months	12 months	12 months
const	0.012***	0.025***	0.017**	0.071**
	(2.84)	(8.83)	(1.98)	(13.85)
$\mathrm{EP}_{i,t}^{\text{recovery}}$	0.451***		1.016***	
	(7.51)		(9.95)	
$\mathrm{EP}_{i,t}^{\text{lower bound}}$		0.837***		0.917***
		(9.27)		(11.79)
adj-R^2(%)	0.64	1.06	1.40	1.62
$H_0 : \alpha = 0, \beta = 1$	0.00	0.00	0.00	0.00
$H_0 : \beta = 1$	0.00	0.07	0.88	0.28
observations	68,815	78,397	56,276	73,715

Panel B. Panel Regression

Dependent variable:	$R_{i,T}^{e}$			
Return Horizon:	6 months	6 months	12 months	12 months
$\mathrm{EP}_{i,t}^{\text{recovery}}$	0.626***		1.232***	
	(8.89)		(11.06)	
$\mathrm{EP}_{i,t}^{\text{lower bound}}$		1.085***		1.126***
		(11.36)		(13.92)
adj-R^2(%)	3.55	4.28	7.08	6.75
$H_0 : \beta = 1$	0.00	0.37	0.04	0.12
observations	68,815	78,397	56,276	73,715

Table 3: The Option-Implied Equity Premium, Firm Characteristics and Subsequent Stock Returns

This table presents panel regressions of realized stock returns on the option-implied equity premium and other firm characteristics. The dependent variable is the realized excess return for the following six months or one year from time t. In each regression, the option-implied equity premium is either $\text{EP}_{i,t}^{\text{recovery}}$ or $\text{EP}_{i,t}^{\text{lower bound}}$ for the corresponding horizon. The regression specification is

$$R_{i,T}^e = \alpha_i + \beta \text{EP}_{i,t} + \gamma X_{i,t} + \epsilon_{i,T}.$$

where $X_{i,t}$ denotes firm characteristics. The characteristics include the log of book value of total assets ($\text{SIZE}_{i,t}$), the book value of leverage ratio ($\text{LEV}_{i,t}$), the market-to-book ratio ($\text{Q}_{i,t}$), the lagged realized returns ($R_{i,t}^e$), and the investment-capital ratio ($\text{INVEST}_{i,t}$). The t-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable:	$R_{i,T}^e$					
Return Horizon:	6 months	6 months	6 months	12 months	12 months	12 months
$\text{EP}_{i,t}^{\text{recovery}}$		0.821***			1.371***	
		(10.80)			(9.46)	
$\text{EP}_{i,t}^{\text{lower bound}}$			1.000***			0.966***
			(9.48)			(9.97)
$\text{SIZE}_{i,t}$	−0.105***	−0.097***	−0.074***	−0.248***	−0.262***	−0.199***
	(-8.69)	(-8.24)	(-7.74)	(-7.74)	(-7.45)	(-8.10)
$\text{LEV}_{i,t}$	0.156***	0.125***	0.0969***	0.336***	0.296***	0.228**
	(3.78)	(3.15)	(2.50)	(3.35)	(2.90)	(2.52)
$\text{Q}_{i,t}$	−0.041***	−0.043***	−0.0155**	−0.085***	−0.090***	−0.037***
	(-8.49)	(-9.34)	(-4.25)	(-8.67)	(-9.16)	(-4.41)
$R_{i,t}^e$	−0.002	0.013	0.040***	-0.018	−0.012	0.026*
	(-0.20)	(1.08)	(3.57)	(-1.40)	(-0.85)	(1.78)
$\text{INVEST}_{i,t}$	−0.194***	−0.183***	−0.152**	−0.267***	−0.329***	−0.274***
	(-3.01)	(-2.95)	(-3.33)	(-2.72)	(-3.20)	(-3.42)
adj-R^2(%)	7.24	8.27	6.61	13.17	14.86	11.23
observations	52,438	52,438	62,534	51,886	43,124	58,894

Table 4: Compatibility of the Two Option-Implied Measures

This table presents the contemporaneous regressions using two option-implied measures of equity premium. In the pooled regression, the specification is

$$\mathrm{EP}_{i,t}^{\text{recovery}} = \alpha + \beta \mathrm{EP}_{i,t}^{\text{lower bound}} + \epsilon_{i,T}$$

where the estimates of equity premium for horizons of six months and one year are used. In the panel regression, the specification is

$$\mathrm{EP}_{i,t}^{\text{recovery}} = \alpha_i + \beta \mathrm{EP}_{i,t}^{\text{lower bound}} + \epsilon_{i,T}.$$

The t-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable:	$\mathrm{EP}_{i,t}^{\text{recovery}}$			
Return Horizon:	6 months	6 months	12 months	12 months
const	0.043***		0.062***	
	(47.63)		(43.58)	
$\mathrm{EP}_{i,t}^{\text{lower bound}}$	1.281***	1.120***	0.684***	0.594***
	(61.25)	(51.22)	(37.21)	(34.99)
fixed effect	No	Yes	No	Yes
adj-R^2(%)	65.99	72.63	54.15	63.16
observations	66,080	66,080	55,911	55,911

Table 5: The Equity Premium and Capital Investments

This table presents panel regressions of capital investments on its determinants. The dependent variable is $\text{INVEST}_{i,t}$, firm i's investment-capital ratio at quarter t. In the regressions, $\text{EP}_{i,t}^{\text{recovery}}$ and $\text{EP}_{i,t}^{\text{lower bound}}$ are the option-implied equity premiums for six-month horizon. The regression specification is

$$\text{INVEST}_{i,t} = \alpha_i + \beta \text{EP}_{i,t-1} + \gamma X_{i,t-1} + \epsilon_{i,T}$$

where $X_{i,t-1}$ denotes the control variables. The controls include the market-to book ratio ($\text{Q}_{i,t-1}$), the log of book value of total assets ($\text{SIZE}_{i,t-1}$), the book value of leverage ratio ($\text{LEV}_{i,t-1}$), return on assets ($\text{ROA}_{i,t-1}$), the value-weighted yields on corporate bonds ($\text{YIELD}_{i,t-1}$), 10-year treasury constant maturity (r_{t-1}^f), and indicator of financial constraints at quarter t including Z-score of cash flow, the WW index, and the KZ index. The t-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable:					$\text{INVEST}_{i,t}$					
Specifications:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\text{EP}_{i,t-1}^{\text{recovery}}$	-0.062^{***}		-0.056^{***}		-0.060^{***}		-0.061^{***}		-0.054^{***}	
	(-3.88)		(-3.55)		(-3.73)		(-3.77)		(-3.08)	
$\text{EP}_{i,t-1}^{\text{lower bound}}$		-0.108^{***}		-0.106^{***}		-0.109^{***}		-0.109^{***}		-0.102^{***}
		(-4.31)		(-4.13)		(-4.27)		(-4.28)		(-3.61)
$\text{EP}_{i,t-1}^{\text{CAPM}}$									0.028^{*}	0.024
									(1.86)	(1.58)
r_{t-1}^f	0.060	0.072	0.059	0.047	0.064	0.054	0.064	0.054	0.078	0.066
	(0.75)	(0.67)	(0.71)	(0.59)	(0.79)	(0.68)	(0.80)	(0.69)	(0.97)	(0.066)
$\text{SIZE}_{i,t-1}$	-0.001	-0.001	-0.002	-0.002	-0.002	-0.001	-0.002	-0.002	-0.004	-0.004
	(-0.44)	(-0.38)	(-0.56)	(-0.52)	(-0.54)	(-0.48)	(-0.75)	(-0.78)	(-1.34)	(-1.27)
$\text{ROA}_{i,t}$	-0.038	0.003	-0.044	-0.016	-0.041	0.002	-0.040	0.001	-0.040	0.0004
	(-1.17)	(0.10)	(-1.35)	(-0.45)	(-1.23)	(0.05)	(0.02)	(-1.04)	(-1.18)	(0.01)
$\text{Q}_{i,t-1}$	0.009^{***}	0.009^{***}	0.009^{***}	0.009^{***}	0.007^{***}	0.009^{***}	0.009^{***}	0.009^{***}	0.009^{***}	0.008^{***}
	(5.81)	(5.56)	(5.48)	(5.27)	(5.70)	(5.43)	(5.65)	(5.39)	(5.15)	(4.86)
$\text{LEV}_{i,t-1}$	-0.023^{*}	-0.028^{*}	-0.021^{*}	-0.022^{*}	-0.023^{*}	-0.022^{*}	-0.023^{*}	-0.023^{*}	-0.026^{*}	-0.025^{**}
	(-1.90)	(-1.91)	(-1.76)	(-1.80)	(-1.82)	(-1.83)	(-1.83)	(-1.85)	(-1.95)	(-1.98)
$\text{YIELD}_{i,t-1}$	-0.079	-0.072	-0.081	-0.073	-0.080	-0.072	-0.079	-0.071	-0.081	-0.074
	(-1.42)	(-1.26)	(-1.43)	(-1.26)	(-1.42)	(-1.25)	(-1.41)	(-1.25)	(-1.40)	(-1.26)
Z-score of $\text{CF}_{i,t-1}$			0.001	0.001^{**}						
			(1.60)	(2.19)						
KZ $\text{index}_{i,t-1}$					-0.0001	-0.0001				
					(-0.30)	(-0.30)				
WW $\text{index}_{i,t-1}$							-0.013^{***}	-0.012^{***}	-0.013^{***}	-0.013^{*}
							(-3.15)	(-3.33)	(-3.15)	(-3.38)
adj-R^2 (%)	49.30	49.45	49.27	49.42	49.25	49.40	49.28	49.45	49.05	49.94
observations	8,653	8,554	8,376	8,287	8,413	8,324	8,408	8,321	8013	7,926

Table 6: The Weighted Average of Cost of Capital and Capital Investments

This table presents panel regressions of capital investments on its determinants. The dependent variable is $\text{INVEST}_{i,t}$, firm i's investment-capital ratio at quarter t. In the regressions, $\text{WACC}_{i,t}^{\text{recovery}}$ and $\text{WACC}_{i,t}^{\text{lower bound}}$ are the weighted average of costs of capital using the option-implied equity premium and the corporate bond yield. The regression specification is

$$\text{INVEST}_{i,t} = \alpha_i + \beta\text{WACC}_{i,t-1} + \gamma X_{i,t-1} + \epsilon_{i,T}$$

where $X_{i,t-1}$ denotes the control variables. The controls include the market-to book ratio ($\text{Q}_{i,t-1}$), the log of book value of total assets ($\text{SIZE}_{i,t-1}$), the book value of leverage ratio ($\text{LEV}_{i,t-1}$), return on assets ($\text{ROA}_{i,t-1}$), and indicator of financial constraints at quarter t including Z-score of cash flow, the WW index, and the KZ index. The t-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable:	$\text{INVEST}_{i,t}$					
Specifications:	(1)	(2)	(3)	(4)	(5)	(6)
$\text{WACC}_{i,t-1}^{\text{recovery}}$	-0.075^{**}		-0.080^{***}		-0.079^{**}	
	(-2.34)		(-2.58)		(-2.57)	
$\text{WACC}_{i,t-1}^{\text{lower bound}}$		-0.092^{**}		-0.091^{**}		-0.090^{**}
		(-2.00)		(-1.98)		(-1.97)
$\text{SIZE}_{i,t-1}$	-0.003	-0.003	-0.003	-0.003	-0.002	-0.003
	(-0.84)	(-0.77)	(-0.85)	(-0.75)	(-0.54)	(-0.96)
$\text{ROA}_{i,t}$	-0.050	-0.013	-0.046	0.012	-0.044	0.011
	(-1.22)	(-0.34)	(-1.07)	(0.33)	(-1.03)	(0.31)
$\text{Q}_{i,t-1}$	0.010^{***}	0.009^{***}	0.010^{***}	0.010^{***}	0.010^{***}	0.010^{***}
	(5.56)	(5.45)	(5.82)	(5.68)	(5.77)	(5.65)
$\text{LEV}_{i,t-1}$	-0.029^{**}	-0.027^{**}	-0.021^{*}	-0.027^{**}	-0.031^{***}	-0.027^{**}
	(-2.44)	(-2.17)	(-1.76)	(-2.25)	(-2.58)	(-2.25)
Z-score of $\text{CF}_{i,t-1}$	0.001^{**}	0.001^{***}				
	(2.07)	(2.99)				
KZ $\text{index}_{i,t-1}$			-0.0001	$-8 \cdot 10^{-5}$		
			(-0.29)	(-0.23)		
WW $\text{index}_{i,t-1}$					-0.0126^{***}	-0.012^{***}
					(-3.35)	(-3.24)
adj-R^2 (%)	48.91	48.94	48.86	48.90	48.90	48.94
observations	8,376	8,287	8,413	8,324	8,408	8,321

Table 7: The Projection of Investment on Equity Premium and Subsequent Stock Returns

This table presents panel regressions of stock returns on the projection of investment on the equity premium and other control variables. The dependent variable is the realized excess stock returns for the period from date t over the next six month. In the regressions, $\mathbb{P}_{\mathrm{EP}_{i,t-1}}\mathrm{INVEST}_{i,t}$ is the projection of investment onto the equity premium. $\mathbb{R}_{\mathrm{EP}_{i,t-1}}\mathrm{INVEST}_{i,t}$ is the residual of investment after the projection. The regression specification is

$$R^e_{i,T} = \alpha_i + \beta_1 \mathbb{P}_{\mathrm{EP}_{i,t-1}}\mathrm{INVEST}_{i,t} + \beta_2 \mathbb{R}_{\mathrm{EP}_{i,t-1}}\mathrm{INVEST}_{i,t} + \gamma X_{i,t} + \epsilon_{i,T}$$

where $X_{i,t}$ denotes the control variables. The controls include the log of book value of total assets ($\mathrm{SIZE}_{i,t}$), the book value of leverage ratio ($\mathrm{LEV}_{i,t}$), the market-to-book ratio ($\mathrm{Q}_{i,t}$), the lagged realized returns ($R^e_{i,t}$). The t-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable:		$R^e_{i,T}$		
Specifications:	(1)	(2)	(3)	(4)
$\mathbb{P}_{\mathrm{EP}^{\mathrm{recovery}}_{i,t-1}}\mathrm{INVEST}_{i,t}$	-12.063^{***}		-10.608^{***}	
	(-6.90)		(-5.88)	
$\mathbb{R}_{\mathrm{EP}^{\mathrm{recovery}}_{i,t-1}}\mathrm{INVEST}_{i,t-1}$	-0.179^{***}		-0.165^{***}	
	(-3.21)		(-3.03)	
$\mathbb{P}_{\mathrm{EP}^{\mathrm{lower\ bound}}_{i,t-1}}\mathrm{INVEST}_{i,t-1}$		-14.661^{***}		-11.819^{***}
		(-10.20)		(-7.65)
$\mathbb{R}_{\mathrm{EP}^{\mathrm{lower\ bound}}_{i,t-1}}\mathrm{INVEST}_{i,t-1}$		-0.176^{***}		-0.160^{***}
		(-3.15)		(-3.02)
$\mathrm{SIZE}_{i,t-1}$			-0.098^{***}	-0.088^{***}
			(-10.60)	(-9.42)
$\mathrm{LEV}_{i,t-1}$			0.162^{***}	0.146^{***}
			(3.49)	(3.20)
$\mathrm{Q}_{i,t-1}$			-0.023^{***}	-0.021^{***}
			(-4.18)	(-4.00)
$R^e_{i,t}$			-0.020	-0.028^{**}
			(-1.59)	(-2.23)
adj-R^2 (%)	2.58	3.71	5.36	5.99
observations	70,435	69,976	70,280	69,749

A Derivation of Lower Bound on Equity Premium

First, I derive the lower bound on the market equity premium following Martin (2017). From the Euler equation, the risk neutral variance of the market return is

$$\mathrm{Var}_t^*\left(R_{m,T}\right)=E_t^*\left(R_{m,T}^2\right)-\left(E_t^* R_{m,T}\right)^2=R_{f,T} E_t\left(M_T R_{m,T}^2\right)-R_{f,T}^2. \tag{25}$$

This risk-neutral variance is related to the market equity premium as follows:

$$\begin{aligned} E_t\left(R_{m,T}\right)-R_{f,T} &= \left[E_t\left(M_T R_{m,T}^2\right)-R_{f,T}\right]-\left[E_t\left(M_T R_{m,T}^2\right)-E_t\left(R_{m,T}\right)\right] \\ &= \frac{1}{R_{f,T}} \mathrm{Var}_t^*\left(R_{m,T}\right)-\mathrm{Cov}_t\left(M_T R_{m,T}, R_{m,T}\right). \end{aligned} \tag{26}$$

Hence, if the negative correlation condition holds, $\mathrm{Var}_t^*\left(R_{m,T}\right)/R_{f,T}$ becomes the lower bound on the market equity premium.

Next, I show that the lower bound can be computed using call option prices. In the derivation, the identity $S_T^2=2\int_0^{\infty}\max(0,S_T-K)dK$ is used.

$$\begin{aligned} \frac{1}{R_{f,T}} \mathrm{Var}_t^*\left(R_{m,T}\right) &= \frac{1}{S_t}\left[\frac{1}{R_{f,T}} E_t^*\left(S_T^2\right)-\frac{1}{R_{f,T}}\left(E_t^*\left(S_T\right)\right)^2\right] \\ &= \frac{1}{S_t}\left[2\int_0^{\infty} \frac{1}{R_{f,T}} E_t^*\left(\max(0, S_T-K)\right) dK-\frac{1}{R_{f,T}} F_T^2\right] \\ &= \frac{1}{S_t}\left[2\int_0^{\infty} \mathrm{Call}_T(K) dK-\frac{1}{R_{f,T}} F_T^2\right] \end{aligned} \tag{27}$$

Now I turn to the equity premium for a stock. The derivation that follows is from Martin and Wagner (2016). Consider a portfolio that consists of tradable assets $\left[R_{n,T}\right]_{n=1}^N$ and generates the maximum expected log return. Let $\left[g_n\right]_{n=1}^N$ denote the weights on asset in that portfolio. To find the weights, I solve the following problem:

$$\max_{\left[g_n\right]_{n=1}^N} E_t\left(\log \sum_{n=1}^N g_n R_{n,T}\right) \quad \text{such that} \quad \sum_{n=1}^N g_n=1. \tag{28}$$

The first order conditions are

$$E_t\left(\frac{R_{k,T}}{\log \sum_{n=1}^N g_n R_{n,T}}\right)=\psi \quad \text{for all } k. \tag{29}$$

It can be proven that $\psi = 1$, by multiplying each of the first order condition by g_k and summing over k. The result indicates that the return on that portfolio, $R_{g,T} \equiv \sum_{n=1}^{N} g_n R_{n,T}$, is the reciprocal of the stochastic discount factor.

Consider a date-T payoff $X_T = R_{k,T} R_{g,T}$. The price of this payoff is

$$E_t \left(\frac{1}{R_{g,T}} R_{k,T} R_{g,T} \right) = \frac{1}{R_{f,T}} E_t^* \left(R_{k,T} R_{g,T} \right). \tag{30}$$

Using the identity that $R_{k,T} R_{g,T} = \frac{1}{2} \left(R_{k,T}^2 + R_{g,T}^2 \right) - \frac{1}{2} \left(R_{k,T} - R_{g,T} \right)^2$, the above price becomes

$$E_t \left(R_{k,T} \right) = \frac{1}{2R_{f,T}} E_t^* \left[R_{k,T}^2 + R_{g,T}^2 - \left(R_{k,T} - R_{g,T} \right)^2 \right]. \tag{31}$$

The RHS can be expressed in terms of the risk-neutral variance of returns, using the fact that $E_t^* \left(R_{k,T} \right) = R_{f,T}$ for any asset.

$$\begin{aligned} E_t \left(R_{k,T} \right) - R_{f,T} &= \frac{1}{2R_{f,T}} E_t^* \left[R_{k,T}^2 - \left(E_t^* \left(R_{k,T} \right) \right)^2 + R_{g,T}^2 - \left(E_t^* \left(R_{g,T} \right) \right)^2 - \left(R_{k,T} - R_{g,T} \right)^2 \right] \\ &= \frac{1}{2R_{f,T}} \left[\mathrm{Var}_t^* \left(R_{k,T} \right) + \mathrm{Var}_t^* \left(R_{g,T} \right) - E_t^* \left[\left(R_{k,T} - R_{g,T} \right)^2 \right] \right] \end{aligned} \tag{32}$$

Dividing the above by $R_{f,T}$, I obtain

$$\frac{E_t \left(R_{k,T} \right) - R_{f,T}}{R_{f,T}} = \frac{1}{2} \mathrm{Var}_t^* \left(\frac{R_{k,T}}{R_{f,T}} \right) + \mathrm{Var}_t^* \left(\frac{R_{g,T}}{R_{f,T}} \right) - \frac{1}{R_{f,T}^2} E_t^* \left[\left(R_{k,T} - R_{g,T} \right)^2 \right] \tag{33}$$

Martin and Wagner (2016) assumes that the last term in RHS is small and has the following structure:

$$-\frac{1}{R_{f,T}^2} E_t^* \left[\left(R_{k,T} - R_{g,T} \right)^2 \right] = \alpha_k + \lambda_t \quad \text{and} \quad \sum_{k=1}^{N} \alpha_k w_k = 0 \tag{34}$$

where $w_{k,t}$ is the weight on stock k in the stock market index. Next, I multiply equation (33) by w_k and sum over k. The result is then

$$\frac{E_t \left(R_{m,T} \right) - R_{f,T}}{R_{f,T}} = \frac{1}{2} \sum_{k=1}^{N} w_{k,t} \mathrm{Var}_t^* \left(\frac{R_{k,T}}{R_{f,T}} \right) + \mathrm{Var}_t^* \left(\frac{R_{g,T}}{R_{f,T}} \right) + \lambda_t \tag{35}$$

Finally, I can relate the expected return on stock k to the market return by subtracting equation (35) from equation (33). That is,

$$\frac{E_t\left(R_{k,T}\right) - R_{f,T}}{R_{f,T}} - \frac{E_t\left(R_{m,T}\right) - R_{f,T}}{R_{f,T}} = \frac{1}{2}\mathrm{Var}_t^*\left(\frac{R_{k,T}}{R_{f,T}}\right) - \frac{1}{2}\sum_{k=1}^{N} w_{k,t}\mathrm{Var}_t^*\left(\frac{R_{k,T}}{R_{f,T}}\right) + \alpha_k. \tag{36}$$